Printed in Japan

ISBN 92-808-0161-9
ISSN 0379-5764

HSDRGPID-47/UNUP-161

AGGREGATED SOCIAL INDICATORS AND SOME THEORETICAL REQUIREMENTS

The Romanian Team

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PRINTED ON RECYCLED PAPER

CONTENTS

These papers by various members of the Romanian Team were presented at the GPID III Meeting, Geneva, 2-8 October 1978 and its continuation in Bucharest, 15-18 January 1979. They can be considered as a contribution to the Indicators sub-projects of the GPID Project.

Geneva, March 1980 — Johan Galtung

It is being circulated in a pre-publication form to elicit comments from readers and generate dialogue on the subject at this stage of the research.

INTRODUCTION

> An indicator is exactly that: something that indicates like a finger, an index pointing to the right direction. [J. Galtung, D. Poleszynski, and A. Wirak, "Indicators for Development" (GPID/UNU paper)]

The present work has four parts, written by three members of the Romanian GPID/UNU team: Gheorghe Păun, Titus Priboi, and Monica Tătărâm. They are in line with the present movement towards the identification of the difficulties — and of the ways to solve them — in defining and constructing adequate and reliable indicators.

The discussions involve the global properties of indicators and hence a high level of generality, as well as some particular examples chosen mostly from the social field. Particular attention is paid to the characterization of multidimensional situations or entities.

The first part, by Gheorghe Păun, "Possibilities and Limits in Reducing the Social Indicators Complexity," is not concerned with criticism of the existing indicators' flaws, but with the reasons for these flaws, reasons that are, among others, responsible for the difficulties — and sometimes the impossibility — of removing these flaws. The author is referring to multidimensional situations that must be evaluated by multidimensional systems of indicators. He has obtained an unexpected result — the impossibility theorem — regarding the aggregated indicators and, by also reviewing the weaknesses and risks of selection, he draws a very significant conclusion with respect to the alternative: will one have to search for small systems of indicators or will one have to concentrate better upon handling complex systems of indicators?

The second part, also by Gheorghe Păun, "An Inherent Restriction on Development Indicators Packages (About Optimal Groupsize)" starts from Yona Friedman's concept of critical groupsize. The strong links that connect the contradiction principle, formulated in this part, to the impossibility theorem, formulated in the previous part, are quite obvious. Let us take an entity characterized by some size — parameters (which can also be viewed as indicators); and by some quality — variables. Hence, we shall have, no doubt, an aggregated indicator of the given entity. But this indicator must be a "good" indicator, so it must be sensitive, stable, and noncompensatory. Or the impossibility theorem tells us that there is no aggregated and simultaneously sensitive, anticatastrophic and noncompensatory indicator. So, the only thing that our aggregated indicator can give up is its noncompensatory character. Or this is precisely what the contradiction principle claims: "When a given entity is characterized by some size — parameters; and some quality — variables, a size and two quality-variables can be found, such that the two variables contradictorily depend on the size."

Further, on the basis of this principle, the following somehow intuitive assertion: "There are no 'good' sizes arbitrarily large . . . any action intended to improve something has at least one negative consequence" — is not only formally stated, but also logically substantiated and appropriately applied to some particular examples.

The third part, by Monica Tătărâm, is "On the Cohesiveness Indicator of Social Groups." It represents another utilization of the Yona Friedman concept of critical groupsize, different from that given to it by Păun in the previous part. While the critical groupsize concept has led Păun to a general investigation of the indicators' global properties, in the present part the same concept has been used in order to improve a given indicator, that of the social cohesiveness within a community. The conclusion obtained joins, in a way, the ideas formulated by Păun, as the author claims that, provided the community — formed by a single social group — stays under the critical groupsize, its social cohesiveness stays above zero.

The social cohesiveness indicator investigation performed in this part starts with a critical discussion of Mario Bunge's definitions of the notions of indicator, social indicator, reliable social indicator and, in particular, of the definition of a social cohesiveness indicator. It offers some new definitions of these notions and, in particular, a new definition of the social cohesiveness indicator. And by doing that, it illustrates the truth of the following assertion by Mario Bunge: "There are not rules of inventing anything, in particular sensitive social indicators; the most one can do is to propose and to discuss examples" (Bunge, M., "What Is a Quality of Life Indicator?" Social Indicators Research, 2, 1975).

The last part is by Titus Priboi, "The Relevance of the 'Electre' Method in Studying the Quality of Life." The author places himself both in the line of the first two papers, by investigating the possibilities of characterizing multidimensional situations and entities, and in the line of the third paper, by giving a new method of constructing "good" and "better" social indicators.

With respect to the second direction of investigation, one notion is to be stressed, that of evaluating a social aspect by means of questionnaires and then of interpreting the answers with respect to a given norm. In this way, an objective-subjective indicator is obtained (as the answers are subjective while the norm is objective).

Further, the importance and the possibilities of investigating multidimensional social situations are revealed. The use of compensatory models and of compensatory methods in the study of these situations is discussed with respect to the impossibility theorem given by Păun. Particular attention is paid to the situations characterized by dichotomic variables.

I. POSSIBILITIES AND LIMITS IN REDUCING THE SOCIAL INDICATORS COMPLEXITY

Gheorghe Păun

1. Introduction*

It is quite redundant to speak about the importance of the socio-economic indicators. In the last century all economists thought and wrote in terms of (economic) indicators. More recently one can speak about a true "social indicator movement" which is expected to clarify and help to solve the great problems of today's world — under- and over-development, the energy crisis, pollution, etc. (See, for instance, the purposes and the results of the World Indicators Programme.)

On the other hand, one may speak of a true crisis of indicators. The insufficiency of economic indicators has become clear. To think only of and to observe only the economic indicators, in other words to be interested only in problems of economic growth, is a quite risky and dangerous position, with disastrous social, psychological, ecological, and political consequences. Let us consider, for example, one of the most used and, lately, one of the most defamed economic indicators, namely the Gross National Product (GNP). Although so largely used, it has serious weaknesses (see Valaskakis, Martin, 1978).

As for the social indicators, the reaction to them is "one of cold stares or open hostility" (Mills, 1978). The reasons are very

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* The present paper tries to incorporate some of the very interesting remarks Johan Galtung, Patrick Healey, Solomon Marcus, Taghi Farvar have made about an earlier version presented at the GPID meeting held in Bucharest, January 1979. Many subsequent useful discussions with S. Marcus and Mihai Botez are also acknowledged.

different: the difficulties (sometimes, the impossibility) to quantify, to numerically evaluate the social entities (Boetz et al., 1978: "the identification of a specific type of measure associated to the concept of quality of life is still a controversial issue"), the incompleteness and the weakness of the existing systems of indicators (C.A. Mills, at GPID Meeting, Geneva, October 1978: "Indicators themselves leave so much out"; K. Valaskakis, at the same meeting: "Today's indicators either indicate too much or nothing"), the subjective (non-objective) character of many social indicators (the quality of life — QOL — of an individual directly depends on the desires-satisfaction; see Mallmann, 1978, Masini, 1978, UNESCO, 1977, etc.).

The difficulties and the disputations related to the indicators problem continue in spite of the great amount of research done and the large number of papers written in this field. (In fact, many papers only criticize existing indicators while others debate general problems, at a low operational level.) The present paper is added to these papers with the ambition to introduce a new point of view in approaching the indicators problem. In short, its purpose is not to investigate the weaknesses of the existing indicators or to propose new ones, but to answer the question, Why have these weaknesses appeared and why haven't they been yet solved? We try to solve the problem taking into account the global properties of the indicators, the requirements which naturally arise when we construct an indicator. Our point of view is mathematical. In this frame we infer that the construction of an indicator system, of a small set of good indicators, is a very difficult task and in some circumstances (see the impossibility theorem in section 10) the problem has no solution (there is no indicator fulfilling three "natural" conditions defining the "good" indicators).

Although assuming a negative form, our results can be considered "positive" in their consequences. The non-existence of an indicator having some given properties indicates a "closed route" as in the circle quadrature problem. There is no aggregated and, simultaneously, sensitive, anticatastrophic (stable) and noncompensatory indicator.

Some critiques of GNP refer exactly to some similar requirements.

The degree of generality of the following considerations is very high. The indicators are viewed as mappings. We are dealing with the global properties of these mappings irrespective of their content in a given real situation. Thus we do not distinguish between economic, ecological, or social indicators, between means and goal indicators, etc. Such an integrating, qualitative investigation seems to be a fruitful one and answers some implicitly formulated requests in the literature: "The politics of indicators (sub-project) . . . should include the following considerations: . . . (2) what barriers and constraints may be anticipated in respect of producing and disseminating new social indicators . . . (3) how may the indicators' wider analysis be used and interpreted" (Cole, Miles, 1978, our emphasis).

2. Indicators, Goals, Processes

No indicator (irrespective of its type) exists out of the triad: goal-process-indicator. "An indicator is nothing more, nothing less than some thing that tells us whether we — a person, a group, a country, a region, the world — are on the right way or not. An indicator is exactly that: something that *indicates*, like a finger, an *index* pointing in the right direction" (Galtung, et al., 1978). "Each social indicator represents a goal, openly or in disguise" (Galtung, 1977a).

Some further remarks about the dialectics of the previous triad. The conscious building of a goal, at any level, whether for an individual, a group, etc., generally has two main steps:

1. the primary formulation of the goal, at a very imprecise level, as a necessity which has become conscious (we call it a *necessity-goal*).
2. the formulation of the goal in concrete terms by means of suitable indicators (we call it a *target-goal*). The necessity-goals are not operational, we can never say that they were or were not accomplished, whether "we are on the right way or not" with respect

to these goals. We must "defuzzify" these primary goals and operationally formulate them, quantitatively if possible, as levels, values to be touched. These levels cannot be defined other than by means of indicators.

After formulating a target-goal we must plan, choose, or consciously describe the processes which drive us to this goal. These processes may be prior to the goal formulation or may be explicitly planned in order to reach the goals. Therefore, the indicators can appear before or after the process starts, when a process is related to a goal. Another situation which can occur is that of "natural" processes, to some extent independent of or even contrary to man's will. Although they are not related to a goal in the above sense, such processes can be side-effects of other goal-oriented processes and we are interested in changing, counteracting them (the necessity-goal): hence we need indicators which evaluate them (in order to be able to define target-goals). Such processes are the ecological ones, pollution and other similar "natural" processes.

Let us point out now another term essentially associated to the triad goal-process-indicator, namely the operation of comparison. As Jonathan Swift teaches us, "Things are great and small only by comparisons." We can know that "we are on the right way" only by comparing the achievements to the targets. Also, the dynamics of a phenomenon are revealed only by comparing the values of the associated indicators at suitable moments. Synchronic comparisons are also needed in order to compare and classify processes or subjects (individuals, groups, etc.).

We want to underline two ideas in the above discussions:

1. The indicators are (almost) always used in order to compare (the goal and the results, the processes, the subjects, etc.).
2. In many situations, the indicators are action oriented: suitable processes are planned in order to modify the values of indicators towards a given goal.

Significant examples of necessity-goals which then are "defuzzified" into target-goals can be found in Mills (1978): "Given the goals of autonomous self-reliant development and the alleviation of the human condition in Africa, we are led to identify as social indicators: (1) Disengagement from the world capitalist system; (2) Democratization of the process of production (that is, social control over the process of social production); and (3) Increasing the productivity of labour as a major concern of the economic planning process." Obviously, "the goal of autonomous self-reliant development," "the alleviation of the human condition," and the "indicators" 1, 2, 3 are necessity-goals, with a high level of generality, unmeasurable hence uncomparable. Such goals cannot be said to be accomplished since we cannot precisely state them. The above goals are then specified in concrete terms by means of indicators of the following type: "(1a) The degree of foreign control over the domestic production process; (1b) The value of exports without a domestic base as a percentage of total exports, . . . (3a) The percentage of unemployment," etc.

3. The Multidimensionality of Socio-economic Indicators

Ideas 1 and 2 in the above section are both contradictory to the multidimensional character of socio-economic indicators.

Indeed, it is obvious that most psycho-social phenomena have multi-dimensional descriptions. Let us consider, for instance, the issue of QOL (for an individual). The obvious necessity-goal is to increase the level of QOL, to improve it. What indicators/system of indicators are to be used in order to formulate the associated target-goal? Even without a very deep analysis, we can see that the QOL of some individual depends on very many parameters. Thus, at least at the beginning of our analysis we must take into account many dimensions of this notion, hence many indicators corresponding to these dimensions.

Here are some significant statements on this subject. "QOL emerges as an evaluation from its components which define life-styles. . . .

Life style has as many components as man has needs. . . . Hence the QOL may only be considered in objective terms for policy purposes and social justice purposes but it always must be considered as linked to life style as subjective evaluation of satisfaction of needs with a given social character" (Masini, 1978). "QOL has as many components as there are needs in the systems of needs. The personal evaluation of the relation between the desires and the attained satisfiers, need by need, determines the QOL components" (Mallmann, 1978).

Let us count the needs in two well-known lists of needs. In Mallmann, 1977, there is a list of 65 needs classified into 27 classes. The preliminary list of Galtung (1977b) contains 32 needs (some of them divided into 2-3 connected needs).

Consequently, as the QOL has a large number of components we must start with a large system of indicators describing the QOL. Initially we may speak of a true over-multidimensionality of the QOL systems of indicators.

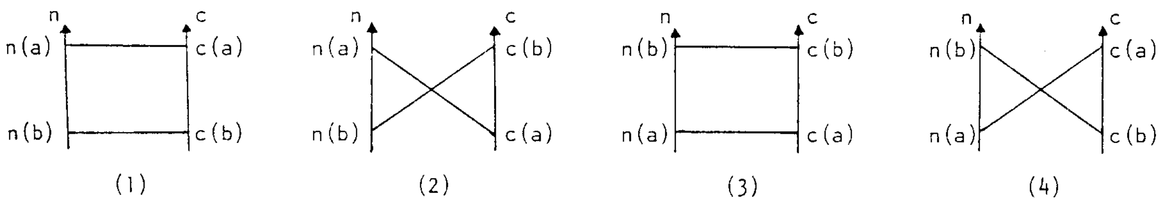


FIGURE 1

4. The Few-Dimensions Necessity

Clearly, the multidimensionality is a disturbing phenomenon. Let us think, for example, of the two previous ideas 1, 2 in section 2. Generally, the comparison of multidimensional entities is a difficult question. How do we define the preference relation between two multidimensional indicators? Consider again the QOL example. Let n and c be two indicators, n evaluating the satisfaction of the need of nutrition and c evaluating the satisfaction of the need of protection against the climate (see Galtung's list). Let a, b be two individuals and let n(a), n(b), c(a), c(b) be the values of the two indicators for the two individuals at a certain moment. The (assumed numerical) values n(a), n(b), and c(a), c(b) can interrelate in various ways (Figure 1).

In cases 1 and 3 it is clear which is the individual having a better QOL, but what can be said about 2 and 4? We cannot compare vectors, even when we deal with two dimensions! What is to be done when we are considering the 65 components associated to the Mallmann's list?

Clearly, from a mathematical point of view we can define order relations on a set of vectors, but these relations do not represent preferences, significant hierarchizations with respect to the QOL problem. For example, we can consider the lexicographic order (we order the vectors observing the first components, then, for non-distinguished vectors, we observe the second components and so on). This solution is not acceptable because it supposes the a priori ordering of the vector components. Such an order is not generally possible (and accepted) for human needs, for instance (see Galtung, Mallmann, etc.).

Neither is the idea 2 in section 2 compatible with multidimensionality. The planning of a process in order to improve the values of a multi-dimensional indicator is, in a sense, an unsolvable problem. Mathematically speaking we are dealing with a multi-objective programming problem: a solution is to be found such that the values of the considered indicators should be simultaneously improved (maximized or minimized). In such a situation the notion of an optimum solution as an optimum solution for all indicators has no sense. In the economic field, such a problem is "solved" in two ways: either the objective functions are aggregated into one function (we will discuss later the troubles which aggregation gives us), or a weaker notion of optimum is used, such as the Pareto optimum.

In the case of social processes, as a consequence of their multi-dimensionality and of the complexity of the corresponding systems, various side-effects can occur, unexpected and, sometimes, undesirable. Sometimes processes started with good intentions (to optimize a certain indicator) have led to bad consequences as side-effects (with respect to other indicators). The greater the number of distinct indicators to be considered, the more difficult it is to forecast all the consequences.

All the above discussion pleads for decreasing the number of indicators in any given concrete situation. The ideal case, when a single indicator is to be considered, clearly solves the previous difficulties completely.

The decrease of the number of (socio-economic) indicators is sometimes very peremtorily requested in literature. For instance in Galtung et al., 1978, it is said: "As the situation is today, we would even say that the more indicators the top élites use, the worse the situation; the number of indicators being a negative indicator." The idea was stressed by Galtung at the GPID Meeting held in Geneva, October 1978: "The more indicators a country has, the worse the situation."

A contradictory situation is to be underlined: the real processes impose large systems of indicators but we need (for many reasons) few-dimensions indicators.

Unfortunately, mathematically speaking, it is a hard task to reduce the number of indicators.

5. Dimension-Reducing Procedures

Basically, there are two ways to reduce the number of components of indicators packages: selection and aggregation.

For instance, let $c_1, c_2, \ldots c_n$ be a multidimensional indicator system for the QOL. Selection means to overlook certain components, to keep others, thus obtaining a fewer-dimensions indicator (Figure 2).

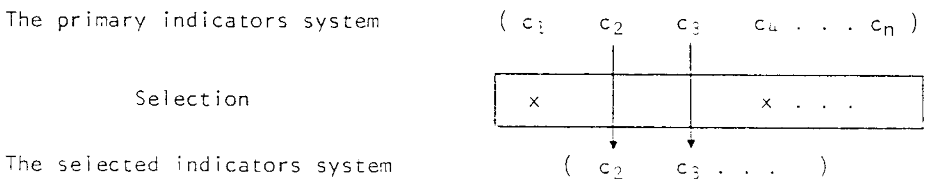


FIGURE 2

The aggregation of some components of the initial system of indicators means to define a unifying mapping which associates one value to a set of values of the initial indicators (Figure 3).

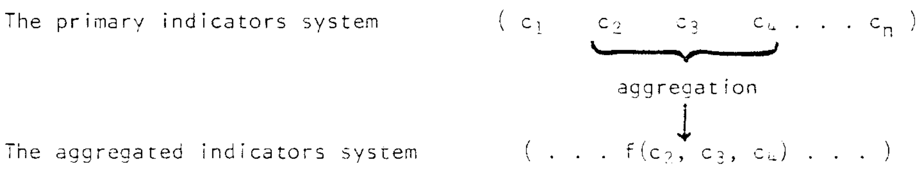


FIGURE 3

Clearly, in a concrete situation, the two possibilities of decreasing the number of dimensions work together. We feel that we must use selection first in order to remove the irrelevant indicators, then, supposing that the retained indicators are mutually independent, we must try to perform aggregations. We shall discuss the two procedures sketched above separately.

6. Selection Criteria and Procedures

There are many methods, more or less sophisticated, that can be used as selection tools. After defining the primary indicators, a logically based preselection is to be performed. Knowing the real process we evaluate by these indicators; we can find redundances, unnecessary parallelisms, irrelevant components. For instance, let us suppose that we have an indicator system aiming to evaluate the QOL of an individual in a country of temperate climate and that this system contains the following two primary indicators: the number of refrigerators per capita and the number of washing machines per capita. We believe that the two indicators are very strongly related, hence one of them is superfluous.

Clearly, such an analysis of the real system is not the mathematician's job. However, important and significant dependencies can be established by using certain mathematical models of the investigated processes. Thus, significant selection can be obtained on statistical bases, that is by looking for redundancies and dependencies in a given set of concrete data related to some given system of primary indicators.

Generally speaking, the statistical data may vary both geographically and temporally. For instance, let us consider the primary indicators

$$i_1, i_2, \ldots, i_n$$

and let us suppose that they are evaluated for the subjects (individuals, groups, countries . . .)

$$s_1, s_2, \ldots, s_p$$

at the moments

$$t_1, t_2, \ldots, t_m$$

Thus we obtain a (n x p x m)-dimensions matrix (Figure 4).

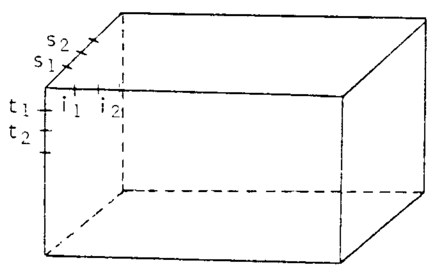


FIGURE 4

In particular circumstances we can know either the values of i_1, i_2, . . ., i_n at a given moment (synchronically) for all subjects or only for a given subject (diachronically) at all the moments t_1, t_2, . . ., t_m. These values, their statistical relationships, may indicate significant relations among indicators. For example, if the values point that two indicators are interrelated by some well-defined function, then naturally one indicator may be ignored, its values being determined by the other. Such functional connections can be found by regression procedures.

For example, in Russett (1964) it is pointed out that the number of radios per 1,000 capita is linearly correlated to the number of newspapers per 1,000 capita. Clearly, a list containing both these indicators is a redundant one.

Let us note that for many phenomena, the linear dependencies cover very many relationships. But, clearly, the linear mappings are not sufficient. In all cases, interesting suggestions about the wanted mapping can be obtained if we first plot the data and look to see if the fit looks substantively linear before adjusting data on the assumption that it ought to be linear.

A similar way is followed by Drăghici (1977). Starting from the values of some indicators for a sequence of moments, a distance is defined between the time-series associated to each indicator. Using cluster-analysis methods, the set of indicators is classified in disjoint classes. For each class one indicator is chosen as representative. The final set of indicators is that of these representative indicators. A completely worked out illustrative example is discussed in Drăghici (1977).

A new way to solve the selection problem is presented in what follows based on an <u>invariance principle</u>. Generally speaking, this principle runs as follows: Let I be a set of information used (in some concrete circumstances) to infer some conclusions C related to some objectives O. If the same conclusions C can be obtained by using a subset I^1 of

I then we say that the passing from I to I^1 is a good selection; it does not entail any loss of information. (The invariance of the set C of conclusions legitimates the selection.)

Obviously, the information in I and the conclusions in C are related to a concrete frame when this information is available and these conclusions are to be obtained. In our case, the input data are the primary indicators values. But . . . to know the results C, this is the whole problem! After knowing C we do not need any selection! Consequently, the invariance principle is applied in the following manner. We specify a subset C_0 of results more easily to be obtained starting from the initial data. Then we validate a selection, using the invariance principle and taking C_0 as the checking set of results. Let I^1 be the selected set of indicators. We extrapolate the invariance from C_0 to C and keep I^1 as a valid selected set used in the next step to infer any conclusion in C.

Clearly, the previously sketched procedure has some risky steps, the main one being the quality of C_0 as a checking set. (A good selection according to C_0 is said to be good according to the goals of the investigations, the whole set of results.) Furthermore, as for any statistical inference, the quality of data, their size, have a sensitive influence upon the quality of results.

As main purposes of indicators use we mentioned the definition of target-goals and the evaluation of processes (either planned or objective). Being fundamental results, they cannot be chosen as validating set C_0 (then the main problems would be solved). The indicators are used also in order to obtain hierarchies and classifications. Such results may be chosen as checking results, particularly since there are well-known methods for multicriterial hierarchization (for instance the "Electre" Method (Roy, 1968), or the methods developed in Onicescu (1970), or of classification (see, for instance, Lerman, 1970). If before and after some selection we obtain the same hierarchy/classification, then the selection is accepted.

However, let us observe that to construct a hierarchy starting from a set of hierarchies defined by individual indicators is exactly an operation of indicator aggregation. The mapping which associates to some object the range in the final hierarchy can be viewed as an aggregating mapping. The impossibility theorem stated in section 10 below says that there is no "good" agregation. Therefore, extending the conditions defining a good aggregation from indicators to hierarchies, it follows that there is no "good" multicriterial hierarchy. Moreover, if a fixed set of objects is hierarchized by using more methods, we frequently obtain more results, a result for each method.

Consequently, we plead for the use of the classification variant instead of the hierarchization one when we chose the checking results.

Finally, let us remark that the above invariance principle as selection-validating tool seems to be (most often unconsciously) used in our daily selections (of parameters, dimensions, components, etc.). However, excepting the trivial cases, a reliable selection cannot be performed without using mathematical tools and often the computer. This is true for the regression analysis, the classifications, the hierarchizations, and the application of the invariance principle itself for a large amount of data.

In many real situations, the selection is performed starting from a set of priorities inferred by people or by decision-makers, political or scientific élites. "I propose that one concentrate on what can be called indicative needs, i.e., needs or group of needs that can be seen to be of key importance. Take, for example, the hunger need — if it is systematic" (Wirak, 1977; a similar idea is formulated in Masini, 1978).

7. The Weaknesses of Selection

Perhaps it is more appropriate to speak about the risks rather than about the weaknesses of selection. Indeed, if the data are carefully chosen, the tools used (mathematical models) are suitable, the results are simulated on real data, then the selection obtained is, statistically speaking, a good one. However, the statistical methods contain intrinsic risks (most often, a priori evaluated). Moreover, the results validation depends on the concrete framework; more exactly, on the concrete goals of the investigation. From a certain point of view, at a certain time, some indicator could be irrelevant and then eliminated. But, from another point of view or at another moment, that indicator might indicate vital phenomena. Thus the quality of a given selection depends on the goals of the investigation and can deteriorate in time as well.

Other risks are connected to the fact that the quality of a situation indicated by an indicator is not directly proportional to the absolute value of the indicator (see further discussion in section 9). For instance, let us consider the following social indicators (Mills, 1978): school enrolment ratios, hospital beds per 1,000 of populations, percentage of households with access to safe drinking water. All these indicators must be counter-balanced by other indicators, since by maximizing their values we are led to a bad situation: more educated unemployment, accentuated rural-urban migration, unutilized and badly staffed hospital facilities, etc. (see Mills, 1978).

8. The Aggregation of Indicators

Let $c_1, c_2, \ldots, c_r$ be some indicators which, by aggregation, give $f(c_1, c_2, \ldots, c_r)$. We consider the indicators c_i to be mappings

$$c_i: \quad P \to R$$

where P is the subject set (individuals, groups . . .) and R is the set of real numbers. For a given subject in P (at a given moment) the primary indicators have the values $c_1(s), \ldots, c_r(s)$ and the aggregated one takes the value $f(c_1(s), \ldots, c_r(s))$. Denoting $h(s) = f(c_1(s), \ldots, c_r(s))$ we obtain a mapping

$$h: \quad P \times P \times \ldots \times P \to R, \quad P \text{ occurs } r \text{ times.}$$

Generally, the aggregation mapping

$$f: R \times R \times \ldots \times R \rightarrow R, \quad R \text{ occurs } r \text{ times},$$

is not a one-to-one function. Indeed, most of the actually used social and economic indicators are not one-to-one. Moreover, "the desirability of indicators of the people, for the people, and by the people" implies the comprehensibility of indicators. Therefore, the analytic expression of each indicator, even aggregated, must be as simple as possible, avoiding sophisticated mathematical notation. But, it is almost obvious that any mapping

$$f: R^n \rightarrow R, \quad n \geqslant 2,$$

which can be expressed by using a bounded number of simple arithematical operations is not a one-to-one mapping (indeed, $a + b = b + a$, $a.b = b.a$, $ca/cb = a/b$ etc.).

Consequently, we may conclude that every aggregated indicator is a many-to-one mapping.

It is apparent, and this can be formally proven (we associate to each value its frequently/probability and we write the entropy of the obtained probability field) that any many-to-one aggregation entails a loss of information. In other terms, a vector $x_1, \ldots, x_n$ determines a unique value $f(x_1, \ldots, x_n)$ but the converse is not true: more vectors can correspond to the same value. If we want to have a synthetic indicator, that is, to obtain a certain type of information, we must pay by information as well.

Many criticisms against the GNP (see, for instance, Valaskakis, Martin, 1978) are based on this remark; indeed, the aggregation up to GNP loses much useful (sometimes, contradictory) information.

9. Mappings Occurring in the Indicators Problem

In this section we survey the mappings occurring in the above discussions. There are three categories of functions (depending on the range set): primary mappings, aggregation mappings, and quality-mappings.

The primary mappings associate quantities (expressed by real numbers) to subjects. For instance, let P be the set of individuals investigated in a problem of QOL. As primary mappings

$$c: \quad P \to R$$

we can consider: the wages, the dwelling surface, the distance from home to the work place, the number of hospital beds per capita, and so on and so forth. In what follows we do not consider further these mappings (they seem to be uninteresting from a mathematical point of view, at least for the purposes of the present paper).

The aggregation mappings are of the form

$$f: \quad R^n \to R, \quad n \geqslant 2.$$

We shall discuss them in the next sections. All these mappings have only an assignment job. They have a neutral character and their values do not necessarily indicate qualities. In order to associate qualities to numerical values we use quality-mappings.

Clearly, these mappings are socio-historically determined and they can express the concrete conditions, the main problems, the priorities of a given society.

Let us think, for instance, of the number of cars per 1,000 of population. Obviously, in no circumstances may the quality of a certain society be considered directly proportional to this indicator. Moreover, in different countries, with different positions regarding the oil problem, this indicator indicates different situations even for similar numerical values.

Anyway, it seems that a quality-mapping

$$q: \quad R \to C$$

(C is an ordered set whose elements denote qualities; we can take c = [0, 1] with the usual order relation between numbers), should have only the following four forms: increasing or decreasing logistic curve, or convex or concave "bell" curve (see Figure 5).

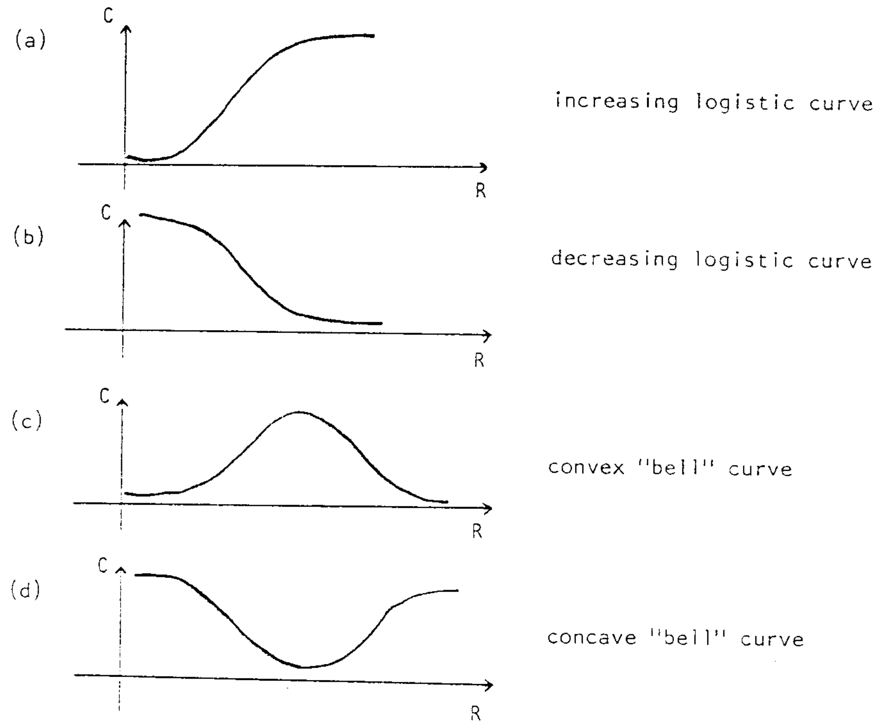
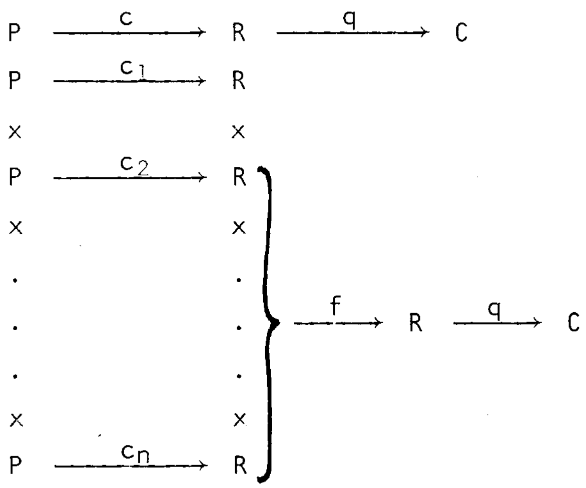


FIGURE 5

Most of the usual indicators seem to have interpretations corresponding to the curve described by Figure 5 (c) (see Galtung et al., 1978), where C is the set of economic development degrees, or the three indicators (from Mills, 1978, discussed in the above paragraph 7, etc.). On the other hand, most negative indicators seem to be of the form in Figure 5 (b). For instance the number of crimes, of suicides, accidents, etc., have such an interpretation. Perhaps there are indicators corresponding to all cases in Figure 5.

We want to emphasize the following idea (in fact, a conjecture): Any quality-mapping can be written as the concatenation of at most two pieces, each of them being monotonous. Moreover, the mappings consisting of two such pieces have large enough monotonicity intervals. (These hypotheses are germane to the next sections.)

In conclusion, we must keep in mind that we are to observe the composition of the above considered mappings in diagrams of the form



For the central intervals of the monotonous pieces of q we can consider that the values of c, respectively f, indicate the associated qualities (non-qualities for decreasing quality-mappings).

10. Three Conditions for the Aggregated Indicators*

Let $f\colon R^n \to R$ and $q\colon R \to C$ and let us take into account only certain interval of monotonicity for q. Therefore we can consider the mapping

$$g = f \circ q : R^n \to C$$

We believe that the following three conditions are naturally to be imposed on the mapping g.

a. The mapping g must be sensitive, that is, it must be monotonous, almost strictly increasing with respect to the positive primary indicators and almost strictly decreasing with respect to the negative indicators. (An indicator is said to be positive if, as in Figure 5 (a), good situations correspond to great values; Figure 5 (b) presents negative indicators.)

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* Sections 10 and 11 incorporate in a modified form (Păun, 1978). The three conditions occurring in that paper are replaced here by weaker assertions.

Let us specify what we mean by "almost increasing" using an example. Let wages be the primary indicator of QOL. For a large interval, this indicator is a positive one. Each increase in wages implies a related increase of QOL. However, with a small increase (by one dollar, for instance), we cannot expect that the QOL of an individual increases (but, clearly, it does not decrease).

Thus, "almost increasing" describes the property that for a significant increase of an argument (the others remain unchanged) the mapping increases but this does not hold for small increases of the argument.

Another natural condition is the following:

b. The mapping g must be <u>stable</u>, that is, for small modifications of the arguments the modifications of the mapping values must be small too. In other words, we request an anti-catastrophic behaviour of g: there should be no jumps of the mapping values.

At least for large intervals, the known socio-economic indicators have such an anti-catastrophic character.

c. Finally, the mapping g must be <u>non-compensatory</u> in the following sense. For each argument we consider a threshold value with the following role. If two vectors in R^n have at least one component differing by a value greater than the threshold, then the mapping g must have significantly different values for these vectors.

In contrast to condition a, the latter conditions refer to the mapping g as a mapping of n arguments. The condition c can be reformulated in the following way: Irrespective of the other arguments, if the values of one of the arguments are very different, then the function values for these arguments are significantly different.

Again, in the frame of the QOL problem let us consider two individuals, the former having much higher wages than the latter but spending much more time shopping and getting from his house to work place. Condition c imposes that the two persons should not have the same QOL; that is, wages do not compensate for the other (negative) indicators.

The compensatory character of GNP is one of the strong points of criticism against it. Let us consider, for instance, "the blindness problem" (Valaskakis, Martin, 1978):

> The GNP indicator measures all activity generated through the market mechanism whether that activity is productive, unproductive, or destructive. The GNP account will give equal place to dollars 1,000 of food-production generated by the agricultural sector and dollars 1,000 paid to workers to dig up holes and fill them up again. . . . An earthquake is "good" because it leads to much reconstruction activity. . . .

It is clear that the aggregation up to GNP loses information and excessively compensates the primary indicators taken into account.

Before writing the next phrase, we want to repeat again the statement: it seems naturally to impose the above three conditions to any (many) aggregated indicators.

The following result, whose proof is given in the next section, makes bootless the efforts towards a "good" aggregated indicator ("good" means "fulfilling the condition a, b, c").

The impossibility theorem. There is no mapping g: $R^n \to C$ fulfilling the above conditions a, b, c simultaneously.

The above theorem seems to have a deep significance. It explains (and authorizes) the dissatisfaction regarding the socio-economic indicators, theoretically proving the impossibility of finding an indicator without drawbacks.

Although negatively formulated, the theorem seems to have very positive consequences. Indeed, it is completely "applicable" (this does not happen with many "positive" results). For instance, it can reassure A.H. Wirak who says:

> Personally, I think and hope that there never will be any possibility of arriving at "the total measurement" of needs satisfaction, either at the individual or at higher levels in society. Only to think that this is possible has an element of horror, as a crime against the nature of man. Compared to that, I would prefer the stupidity of GNP/cap or similar measurements. [Wirak, 1977]

Concerning the existing indicators (and those which will be imagined) it follows that they do not fulfil all the three conditions. Most often condition c is rejected. On the two intervals of monotonicity, any indicator obviously has the property of sensitivity. The anti-catastrophism of the existing indicators is out of any doubt. It remains to sacrifice the non-compensatory demand.

On the other hand, there are many points of view from which the compensatory character of an indicator is not a negative feature; sometimes the opposite assumption is more suitable. Let us think, for example, of a system of indicators related to certain satisfiers (in a problem of QOL). Clearly, the satisfiers compensate each other. A framework in which this idea could be rigorously investigated in connection with the desires, the needs and the illness states of some individual, is that introduced in Mallmann, Marcus, 1978. In this frame an indicator of human desires satisfaction may be compensatory. More particularly, the nutriments compensate each other (the daily need of calories, proteins, etc., can be satisfied by different nutriments). Starting from some primary indicators associated to categories of nutriments, an aggregated indicator of the nutrition quality will be a compensatory one.

A completely aggregated indicator of QOL is advocated in Romalo, 1979, where the request for a compensatory unique indicator is almost directly formulated: "Using the concept of QOL we shall attempt to define a [hence one] measurable quantity which could represent either the degree of well-being or of discomfort an individual feels when living in a given society. . . . There exists a possibility to correlate the sensations within a coherent set." On this basis we are led to a representation of QOL perception "in mathematical form by a real function s of a certain number of variable arguments, $x_1, \ldots, x_n$ and time

$$s = f(x_1, \ldots, x_n, t)."$$

11. The Proof of the Impossibility Theorem

In formulating the conditions a, b, c we used the terms small, significant, great, very great, describing differences, increases, values. We shall consider that these sizes are defined by threshold values suitably chosen for each indicator. For instance, for the primary indicator "wages," a small increase is of no more than, let us say, 50 dollars, a significant one is of 50-100 dollars, a very great one is of at least 1,000 dollars.

In the following discussion we assume the thresholds to be given, the same for all the three conditions. This is the single condition we need about the semantics of the size-values small, significant, etc.: the three conditions a, b, c should refer to the same thresholds, irrespective of their concrete values.

Always, we assume the order relations induced by the sequence small, significant, great, very great.

Now, let us suppose that there is a mapping

$$g: R^n \to C, \quad n \geqslant 2$$

satisfying all the three conditions. For a given

$$\alpha = (z_1, \ldots, z_{n-2})$$

we consider the mapping

$$g_\alpha(x, y) = g(x, y, z_1, \ldots, z_{n-2})$$

Let us suppose that the indicators corresponding to the first two components of the argument of g are of positive type and hence g is almost increasing with respect to these arguments. Therefore, g_α is almost increasing with respect to both its arguments. The case of two negative indicators or of two indicators of different types can be treated in a similar way.

Let us consider two values for x and y sufficiently different so that their difference may be characterized as very great.

Let x_m, x_M, y_m, y_M be these values. We consider the following four mappings

$$g_1(x) = g_\alpha(x, y_m),$$
$$g_2(x) = g_\alpha(x, y_M),$$
$$g_3(y) = g_\alpha(x_m, y),$$
$$g_4(y) = g_\alpha(x_M, y).$$

Let us consider furthermore the following four points

$$a = g_1(x_m) = g_\alpha(x_m, y_m) = g_3(y_m),$$
$$b = g_2(x_m) = g_\alpha(x_m, y_M) = g_3(y_M),$$
$$c = g_4(y_m) = g_\alpha(x_M, y_m) = g_1(x_M),$$
$$d = g_4(y_M) = g_\alpha(x_M, y_M) = g_2(x_M).$$

The values a and d are the extreme values of the mapping g on the intervals (x_m, x_M), (y_m, y_M) and b, c are intermediate points. In order to make clearer the next reasoning, let us consider the graphs in Figure 6.

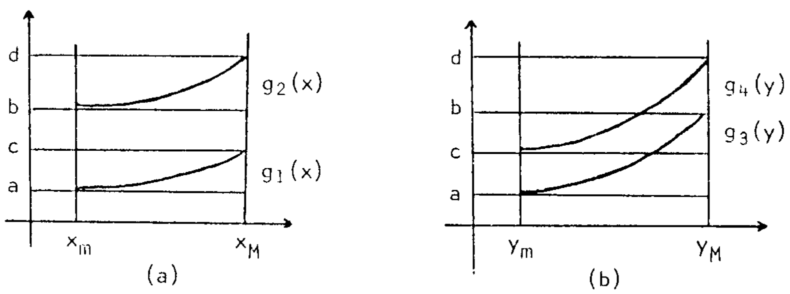

FIGURE 6

The difference between x_M and x_m is very great; the same with that between y_M and y_m. Consequently, in view of condition c, $g(x_m, \alpha)$, $g(x_M, \alpha)$ are significantly different. Two cases are possible: either $b < c$ or $c < b$. Both of them can be similarly treated.

Let us suppose that $b > c$, that is, we have the situation illustrated by Figure 6. Let us focus our attention on Figure 6 (b) where the projections of the two graphs on the vertical axis overlap.

Let y_1, y_2 be two distinct values of y such that

$$g_3(y_1) \leqslant c \leqslant g_3(y_2)$$

that is

$$g(x_m, y_1, \alpha) \leqslant c \leqslant g(x_m, y_2, \alpha).$$

If

$$g_3(y_2) - g_3(y_1)$$

is a small value, then

$$g_3(y_2) - c = g(x_m, y_2, d) - g(x_M, y_m, \alpha)$$

is also a small value. This contradicts condition c. Therefore, the difference between $g_3(y_2)$ and $g_3(y_1)$ must be at least significant. From condition b it follows that the difference $y_2 - y_1$ must be at least significant.

Let us consider the intermediate values of y between y_1 and y_2. From condition a the mapping g_3 (hence g too, for suitably chosen arguments) takes intermediate values between $g_3(y_1)$ and $g_3(y_2)$. All these values must be far (that is, at a significant distance) from c (the condition c). Consequently, there is y_3 between y_1 and y_2 such that for $y < y_3$ the values of g_3 are significantly less than c and for $y > y_3$ the mapping g_3 takes values which are significantly greater than c. (The value for y_3 is also significantly different from c.) Let us consider two values very near (at a small distance) to y_3, one less than y_3 and the other greater than y_3. In view of condition b the values of g_3 for these points must be at a small distance. Contradiction. The existence of the mapping g is impossible.

Remarks. The above proof develops identically when the mapping g is not defined on R^n but on an n-dimensional rectangle. This is a more realistic case. For instance, we can consider the intervals for which the mapping g is monotonous. The single restriction is that the size of these intervals be very great. Moreover, we can work on discrete sets of numbers not necessarily on real intervals. The single restriction is now that the difference between any two consecutive values should be small.

12. Fractionary Indicators

A special category of indicators is that of fractionary indicators. The ascribing of a measurement to another has two purposes: to make objective the first indicator or to evaluate an efficiency indicator. For instance, two countries of different sizes and economic powers cannot be compared by means of the absolute value of GNP. Such a comparison cannot account for the productivity of the two countries. Then, the GNP is related to the population number. On the other hand, in order to know a process which transforms some inputs in some outputs it is not sufficient to know only absolute values about it. We must consider the ratio output/input in order to see the efficiency of the process, its out-turn.

Such indicators are much used in the economic field in any country. The cost-benefit analyses supply many examples. The new economico-financial mechanism promoted in Romania pays great attention to fractionary indicators. Here are some such indicators: the total and the material expenses per 1,000 lei of wares production, the benefits and the net production per 1,000 lei of fixed capital, etc.

Clearly, the fractionary indicators are aggregations and therefore they are compensatory (the conditions a, b seem to be always fulfilled in the economic field). In order to eliminate this shortcoming we must specify one of their two components. At first sight we obtain nothing in this way since we have again two indicators. However the fractionary indicator is more expressive than its separate components, it gives synthetic information.

The fractionary indicators raise interesting mathematical questions.

Let us consider the following problem. A country wants to construct two industrial units. Three proposals are formulated. For each of them the annual imports and exports are known. Clearly we must choose the two units which give the greatest ratio total export/total import.

The above problem has been proposed to about 100 persons with economic concerns (at the level of enterprise). All of them have indicated as solutions of the problem the choice of the two objectives having the greatest ratio export/import.

But let us consider the following example:

Industrial unit	U_1	U_2	U_3
Export	4	2	3
Import	7	4	2

We have the relations

$$\frac{3}{2} > \frac{4}{7} > \frac{2}{4}$$

hence, following the interviewed experts, we have to choose the units U_1 and U_3. However the pair U_2, U_3 gives a better solution. Indeed

$$\frac{2+3}{4+2} = \frac{5}{6} > \frac{4+3}{7+2} = \frac{7}{9}$$

Consequently, the intuition of some experienced persons failed! The explanation of this apparently paradoxical result is obvious: our mind works linearly, it knows how to optimize only linear objective-functions. Here the objective-function was non-linear; linear reasoning was projected on a non-linear problem, and naturally it failed.

The above problem can entail some pessimistic conclusions about the demand for indicators "of the people, for the people, and by the people." The fractionary indicators can be of the people and for the people (they are comprehensible) and can be formulated by the people. However, the choice of processes, the strategies to be followed in order to optimize such indicators cannot be accomplished at the level of intuition. When the problem must be solved there should be a mathematician who doubts the "evidences" of the intuition. (The above problem was mathematically solved by Bǎlutǎ, Pǎun, 1978.)

13. Other Conditions about the Mapping-Indicators

By imposing global conditions on indicators, interesting results could be obtained by means of some known theorems from the mathematical analysis (as S. Marcus has pointed out in an oral communication). In this way one can deduce the analytic form of some indicators, a hard task when it must be done empirically.

For instance, let us impose on an indicator that it must be monotonous and linear. (A mapping f for which $f(x+y) = f(x) + f(y)$ is said to be linear.) One can prove (see Marcus, 1958, and the bibliography given there) that a mapping with these properties must be of the form: $f(x) = ax$, a given.

The same result can be obtained if we want to have a linear and bounded indicator or in other similarly weak circumstances. Similar results can be obtained when we impose the weaker request that the indicator be internal (see Marcus, 1956); a mapping $f: (a, b) \to R$ is internal if for any x, y we have the inequalities

$$\min [f(x), f(y)] \leqslant f\left(\frac{x+y}{2}\right) \leqslant \max [f(x), f(y)].$$

This last condition seems to fit many socio-economic situations.

In a forthcoming paper, such results will be discussed in more detail.

14. Conclusions

Although very general, the approach developed in this paper seems to lead to useful results. Especially the impossibility theorem can explain and justify many criticisms and failures.

The main conclusion of the paper is that there are intrinsic difficulties when we want to decrease the number of indicators describing a given situation. A multidimensional reality must be evaluated by a multidimensional system of indicators. Thus, one can infer the conclusion that it is better to concentrate upon dealing with complex systems of indicators, planning and evaluating processes with respect to such systems, and not upon searches for small systems of indicators.

As a consequence we get that the QOL and other similar multidimensional entities are uncomparable; no good hierarchy can be inferred on this basis.

Finally, let us note the similarity between our impossibility theorem and the theorem of the impossibility of aggregating group decisions (Arrow, 1959). However, there the frame was a logical one and the conditions, completely different from the previous ones, characterized group-decision rationality in completely different terms.

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II. AN INHERENT RESTRICTION ON DEVELOPMENT INDICATORS PACKAGES (ABOUT OPTIMAL GROUPSIZE)

Gheorghe Păun

The purpose of the present paper is to formulate and to illustrate (by means of Yona Friedman's study of critical groupsize and by the case of hierarchical groups) the following principle (called the contradiction one) which seems to us to act in many, perhaps in all, circumstances: When a given (social, economic, etc.) entity is characterized by some size-parameters and some quality-variables, a size and two quality-variables can be found such that the two variables contradictorily depend on the size. In other words, every significant indicators package contains at least two indicators which cannot be simultaneously improved. Such a principle validates the inherent difficulties in optimizations of any type and calls for inherent limits of growth for many (all?) entities.

Clearly, a much more ambitious and important question is to find a proof of the contradiction principle. The present páper only illustrates it. But it would be very interesting to construct a theoretical (mathematical) framework such as, starting from certain given hypotheses, as weak as possible, to infer the contradiction principle; that is, to obtain it as a consequence of given primitive assumptions. The importance of such a result justifies further investigations.

1. Introduction

The starting point of the present paper is Yona Friedman's work on "critical groupsize." More concisely, the following paragraph from Friedman (1978) contains all the initial impulses behind our investigations:

> We will call critical size the limiting size when growth stops and beyond which if an entity continues to grow, the growing entity suffers important qualitative changes. Critical group-size will thus be the size of a social group characterized by certain qualities, over which size this group cannot keep these qualities. Critical groupsize is effectively critical to the good functioning of a group or of an organization.

Certain questions are to be formulated with respect to the underlined words:

a. What and which is the size? Any object, phenomenon, process, etc., should be characterized by a set of parameters. Which of these parameters describes "the size"?

b. How can we delimit an entity? When may a parcel of reality be called an "entity" (in such a way that we may speak about its size and about indicators describing it)?

c. What is a quality? Is the "size" a quality or not?

d. What is a good functioning? Is the "good functioning" opposite to "suffers important qualitative changes" or not?

In the quoted Friedman paper, the answers to these questions are clear: (a) the size of a group is evaluated by the number of persons involved in that group; (b) the entity studied is the group and a group is "a set of individuals in which there exists some sort of 'relations' between any two individuals belonging to the set. A person who has no such relation to at least one other who in his turn is related to the others, can be considered a person 'out of the group'"; (c) the qualities involved are the "valence" and the "channel capacity" (respectively, the maximum number of relations a person can have and the "capacity for transmitting a message with a number of errors"); (d) a group functions well when the valence and the channel capacity thresholds are not surpassed.

Some general ideas are to be pointed out. When approaching our entity (the group in the above case) we must distinguish two types of parameters characterizing it: "sizes" and "qualities," that is, objective, neutral quantities without inherent qualitative interpretations, and parameters of the second order exposing qualities. The number of persons in a group, the number of links in a group are "sizes"; the concrete values of these parameters do not indicate "good" or "bad" groups. The valence and the channel capacity actually realized in a group evaluate its quality (they are parameters of the second type).

However, we cannot say that we are dealing with parameters and indicators since also the "sizes" can be viewed as indicators. Although primarily these parameters do not indicate "qualities," they can however "tell us whether we — a person, a group, a country, a region, the world — are on the right way or not." They can point "like a finger . . . in the right direction" (Galtung et al., 1978). A quality is thus associated to any size when it is used as an indicator for defining target-goals (in the sense of Păun, 1979): a situation is better when it corresponds to a parameter value nearer to the target value.

Moreover — and this is the second idea derived from the above quotation from Friedman — the quality of sizes is evaluated by the quality of second-order parameters. This is the main point of our paper. In many (all?) situations (we illustrate this by the critical groupsize and by the case of hierarchical groups) various second-order parameters associate contradictory qualities to a given size parameter; that is, the growth of the size parameter has contradictory influences upon the qualities indicated by the second-order parameters (some qualities are improved and some decreased when size is increased). Thus — the third idea — we are led to consider the problem of finding the suitable sizes in given circumstances; even when the critical sizes are not touched we must choose economical sizes, eventually the optimal ones.

We conjecture that a sort of contradiction principle is acting in many (all?) socio-economic problems: for any size parameter two situations occur: either the associated quality-parameters decrease when the size increases (they are negative indicators) or there are both negative and positive quality-parameters (but not only positive quality-parameters). In other words, in many (all?) situations at least a negative parameter is to be considered.

Therefore, the situation in Figure 1 is met in many (all?) circumstances.

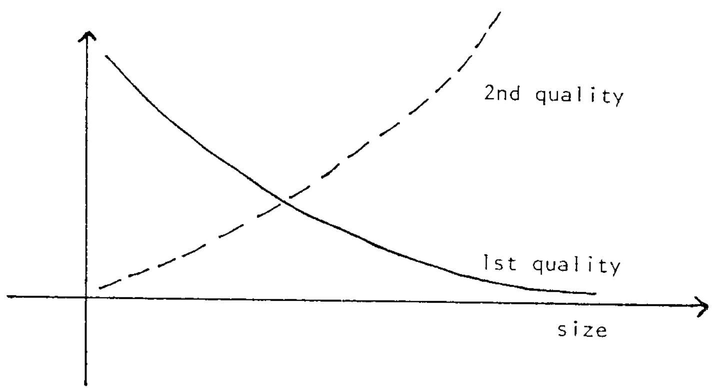

FIGURE 1

Indeed, many apparently "negative" sizes have only negative associated qualities. Let us think of the number of suicides, of crimes, of accidents, of ill men (when considering the quality of life of a nation, for instance), etc.

Conversely, we do not know any "positive" size to which only positive qualities are associated; if such a negative quality is not visible, then the effort to increase the size can be taken as a negative indicator.

Let us recall from Păun (1979) the impossibility theorem of indicators aggregation: There is no aggregation method which is simultaneously sensitive, anti-catastrophic and non-compensatory. The conclusion was that "there are intrinsic difficulties when we want to decrease the number of indicators describing a given situation. A multidimensional reality must be evaluated by a multidimensional system of indicators." The contradiction principle completes the assertion: the systems of indicators are multidimensional and inherently contradictory. These considerations have a practical (unfortunately, negative) significance: when we are acting on certain parameters (sizes) generally we improve some indicators and deteriorate others. The importance of this assertion seems to demand further investigations in order to check whether it is true everywhere or not, and to find ways to minimize its influence.

2. An Illustration of the Contradiction Principle — the Critical Groupsize

Trivial illustrations of the contradiction principle can be observed in daily life. Let us think, for instance, of a vehicle for public transport (a bus, for example). Consider the following three parameters: the number of persons in the vehicle, the comfort of these persons when travelling in this vehicle, and the economy of vehicle activity. The former parameter evaluates a size whereas the others are quality indicators. Clearly, the two qualities contradictorily modify when the number of persons increases.

Significant materializations of the contradiction principle can be found in various mathematical fields. Let us think, for instance, of the trade-off results in computational complexity and in descriptional complexity of formal languages (see Calude, Păun, Simovici, 1980, and the bibliography given there). But note that there we have a somewhat different situation: we cannot simultaneously improve two complexity measures, for instance. In our case we deal with three parameters, two of them — the qualities — contradictorily depending on the third one.

We recall now, in short, some notions from Friedman (1978).

Let us visualize a group by a directed graph where vertices denote persons and arrows indicate influences between persons. Any such graph is a connected one (for any two vertices there is a link — not necessarily a path consisting in directed arrows — between them). Two parameters are defined for such a graph (for the corresponding group). The valence of a graph is the maximum degree of the points in graph (the number of links incident to/from a given point). (The idea of valence appears in Galtung, 1977, in a more "chemical" framework.) The channel capacity of a graph can be defined as the maximum length of the shortest paths between any two vertices in the graph. (The messages transmitted in a group deteriorate, such that the influences between individuals will be indistinguishable beyond a given number of transmissions.)

The size of a graph is characterized by the number of vertices and the number of edges. If we take as constant one of these parameters and consider groups of various sizes according to the other, clearly we obtain that the valence and the channel capacity change in opposite directions with respect to the constant parameter. The assertion can be formally proved. Instead of a rigorous proof we prefer an example.

Consider, for instance, all the groups having four nodes. The graphs associated to these groups can contain 0, 1, 2, 3, . . ., 11, or 12 arrows (12 = 4.3 possible different arrows). The graphs having 0, 1, or 2 edges (and four vertices) are not connected hence do not represent groups. Table 1 contains the associated valences and channel capacities (the smallest values for all graphs having the indicated number of arrows) for the graphs with 3, 4, . . ., 12 arrows.

Figure 2 contains optimum graphs for these situations.

TABLE 1

Arrows	Valence	Channel capacity
3	2	∞
4	2	3
5	3	3
6	3	3
7	4	2
8	4	2
9	5	2
10	5	2
11	6	2
12	6	1

The contradictory evolutions of the two qualitative parameters are obvious.

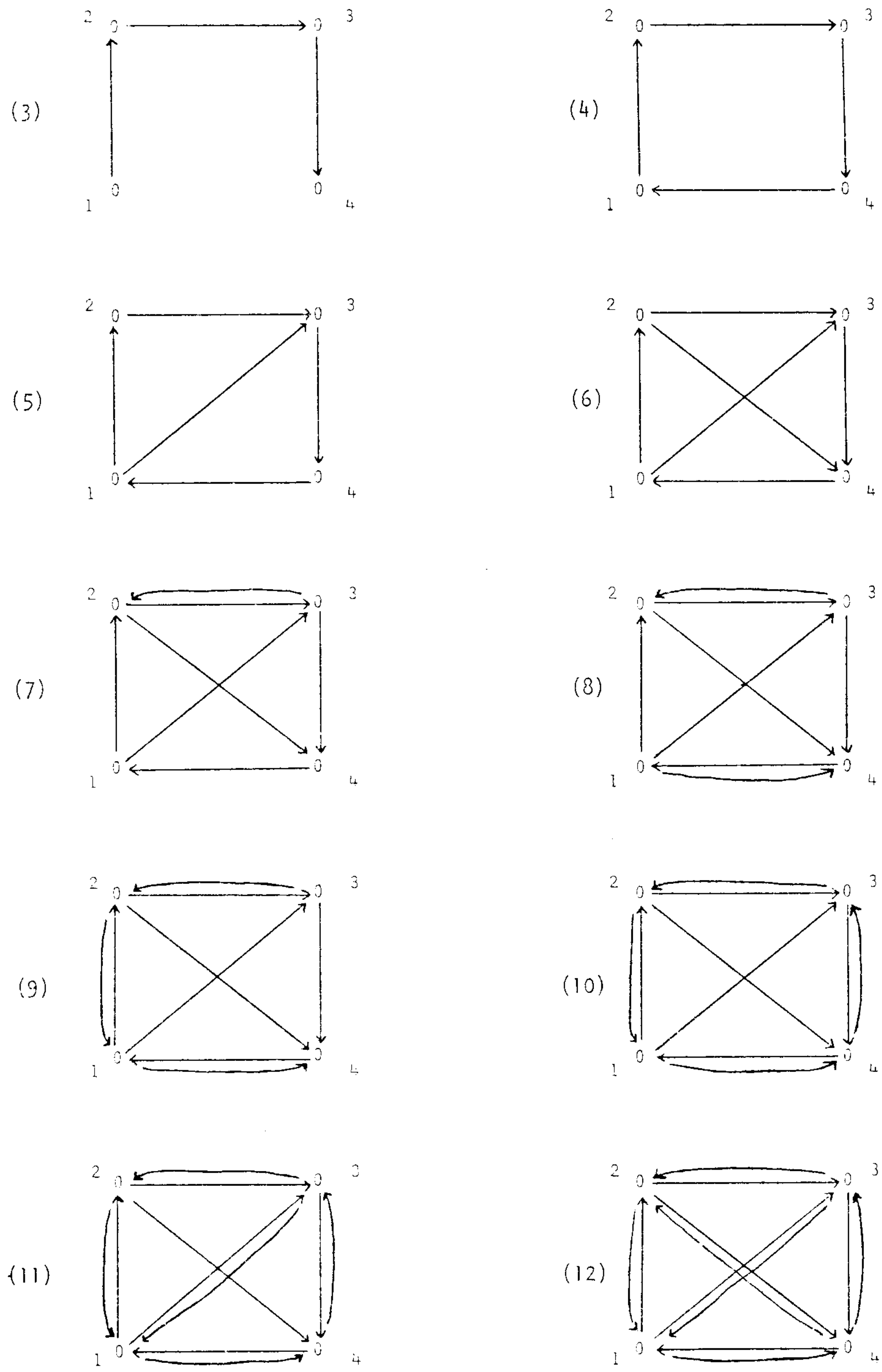

FIGURE 2

3. The Contradiction Principle in Hierarchically Structured Groups

We discuss now a further illustration of the contradiction principle by investigating some parameters related to the quality of hierarchical structures. Besides an illustrative purpose, these considerations seem to have also an intrinsic significance in the study of hierarchical groups, in management, etc.

First, let us remark that many (all?) hierarchically structured groups are characterized as well-defined entities primarily by a common goal and then by the relations between the members of the group. Let us think of economic units, military units, universities, and so on. This is of central importance for the considerations in this section: the existence of a unique goal (formulated either by the environment or by the group itself) that should be accomplished by the group. But the group acts by means of the participants acting. Thus the unique goal of the group must be parcelled in such a way that each element of the group has its own goal, its own trajectory. It is a common fact that the goals are (or must be so) formulated by means of suitable indicators specifying levels, values, thresholds for precise moments. Generally, these moments are more rare for higher levels in a hierarchy. For instance, the enterprise plan can be formulated at the level of the year or of the quarter, but for a section the week or the month should be taken as the interval; for workers the plan must be formulated in daily terms.

Let us consider a group Σ consisting of m subgroups and let us assume that the plan horizon [0, T] from the level of Σ must be divided into n intervals at the level of subgroups. Let us evaluate the complexity of passing from Σ to subgroups (the temporal and geographical parcelling). Intuitively, the parcelling effort depends both on the number n of periods and on the number m of subgroups. Moreover, it seems to be obvious that the effort increases when n and m increase more rapidly than linearly. A dependence as in Figure 3 seems to be met. We propose the following Cobb-Douglas-like formula:

$$\text{Effort}(n, m) = kn^{\alpha}m^{\beta}$$

where k is a positive real coefficient and α, β are real numbers greater than one.

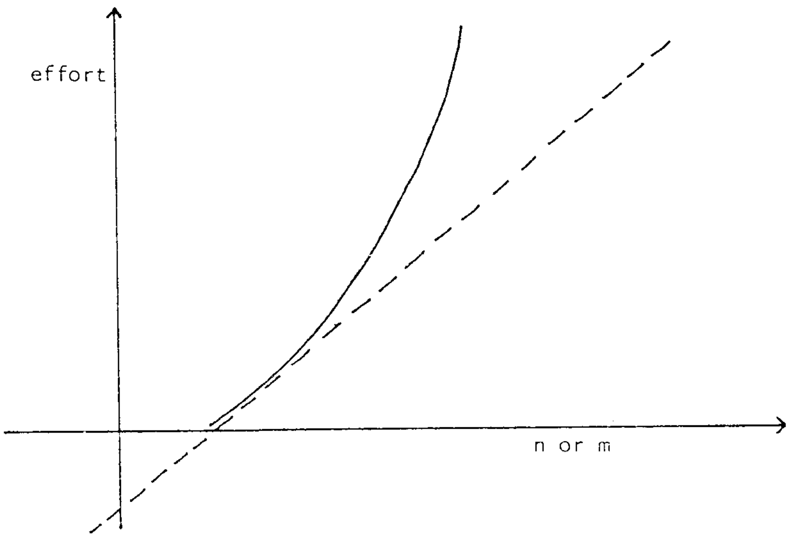

FIGURE 3

A natural problem arises: what happens when the hierarchy increases: that is, what is the cost of parcelling in a higher hierarchy compared to that in a less high one? In other words, how does the parcelling cost depend on the size of hierarchy? The answer will be found in what follows: <u>the hierarchization decreases the total parcelling cost</u>.

Indeed, let us consider a group as above. The parcelling effort is

$$\text{Effort } (n, m) = kn^{\alpha}m^{\beta}, \text{ with given } \alpha, \beta.$$

Consider now a three-level organization of Σ (as in Figure 4).

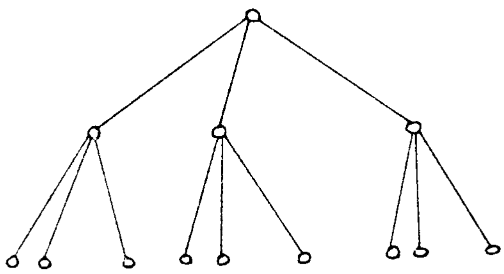

FIGURE 4

We assume that on level 2 we have r subgroups, the i-th subgroups containing m(i) individuals. Consequently

$$2 \leqslant r \leqslant m,$$

$$m \leqslant 3,$$

$$\sum_{i=1}^{r} m(i) = m$$

Suppose also that on level 2 the plan horizon [0, T] was divided into p identical intervals (of length n/p). Therefore we must have

$$2 \leqslant p \leqslant n/2,$$

$$n \geqslant 4.$$

(We considered both geographical and temporal parcellings to be non-trivial.)

The effort to pass from level 1 to level 2 is

$$\text{Effort } (p, r) = kp^{\alpha}r^{\beta}$$

and the passing from one subgroup to its components (in a given period) has

$$\text{Effort } [n/p, m(i)] = k \frac{n^{\alpha}}{p^{\alpha}} m(i)^{\beta}.$$

The total parcelling effort in this hierarchy is

$$\text{TEffort} = kp^{\alpha}r^{\beta} + p \sum_{i=1}^{r} k \frac{n^{\alpha}}{p^{\alpha}} m(i)^{\beta}.$$

We rewrite this equation in the following way:

$$\text{TEffort} = kp^{\alpha}r^{\beta} + pk \frac{n^{\alpha}}{p^{\alpha}} \sum_{i=1}^{r} m(i)^{\beta} = kp^{\alpha}r^{\beta} + k \frac{n^{\alpha}}{p^{\alpha-1}} \sum_{i=1}^{r} m(i)^{\beta}.$$

As $m(i) \geqslant 1$ and $\beta \geqslant 1$ for each i, we have $\sum_{i=1}^{r} m(i)^{\beta} < [\sum_{i=1}^{r} m(i)]^{\beta}$, hence

$$\text{TEffort} < kp^{\alpha}r^{\beta} + \frac{n^{\alpha}}{p^{\alpha-1}} [\sum_{i=1}^{r} m(i)]^{\beta}.$$

As $r < m$, $\beta \geqslant 1$ and $\sum_{i=1}^{r} m(i) = m$, we obtain

$$\text{TEffort} < kp^{\alpha}m^{\beta} + \frac{n^{\alpha}}{p^{\alpha-1}} m^{\beta} = km^{\beta} (p^{\alpha} + \frac{n^{\alpha}}{p^{\alpha-1}}).$$

But $p \leqslant n/2$ and $2 \leqslant p$, hence

$$\text{TEffort} < km^{\beta}\left[\left(\frac{n}{2}\right)^{\alpha} + \frac{n^{\alpha}}{2^{\alpha-1}}\right] = km^{\beta}\left(\frac{n^{\alpha}}{2^{\alpha}} + 2\frac{n^{\alpha}}{2^{\alpha}}\right) = km^{\beta}\, 3\frac{n^{\alpha}}{2^{\alpha}} = \frac{3}{2^{\alpha}}\text{Effort }(n, m)$$

For $\alpha \geqslant \log_2 3$ we obtain $\frac{3}{2^{\alpha}} \leqslant 1$, hence

$$\text{TEffort} < \text{Effort }(n, m).$$

Consequently, for $k \geqslant 0$, $\beta \geqslant 1$, $\alpha \geqslant \log_2 3$ we obtain that the parcelling is strictly easier in the three-level hierarchy than in the two-level one. The size of the hierarchy has a positive influence on the parcelling cost diminishing it. (In the Annex we obtain a similar result for geographical parcellation only taking the effort formula.)

$$\text{Effort }(m) = km^{\beta},\ k \geqslant 0,\ \beta \geqslant 2.$$

But, the contradiction principle is on duty! The hierarchy implies tension and compression (Judge, 1978), a high degree of bureaucratization, rigidity, etc., etc. (Remember all the shortcomings of β-structures compared to α-structures.) Moreover, the inverse passing, from individuals to the entire group, the aggregation, is disadvantageous when the hierarchy is high. Let us think of the loss of information entailed by such an aggregation and the necessarily compensatory character of it (Păun, 1979). From these points of view, <u>the higher the hierarchy, the worse the situation</u>. The contradiction is obtained.

4. Conclusions

Clearly, the subtitle of the present paper is too ambitious. It is a hard task to find the optimum size of an entity. In fact, taking into account the multidimensionality of the evaluating quality parameters, the notion of an optimum situation cannot be easily defined; special kinds of optimality — such as the Pareto optimum — must be considered since we cannot generally look for an optimum solution according to more than one objective function. However, some obvious conclusions can be derived from the above discussions. Let us look again at

Figure 1. In practical situations precise thresholds are given indicating admissible values for the quality parameters. Such thresholds are used in Friedman (1978) in order to define the critical groupsize. These thresholds determine intervals for which the corresponding quality-paramc rs have "good" values. Let us consider Figure 5 where such intervals are visualized.

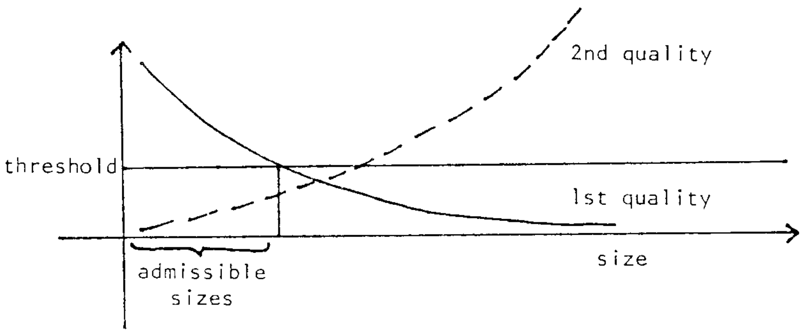

FIGURE 5a

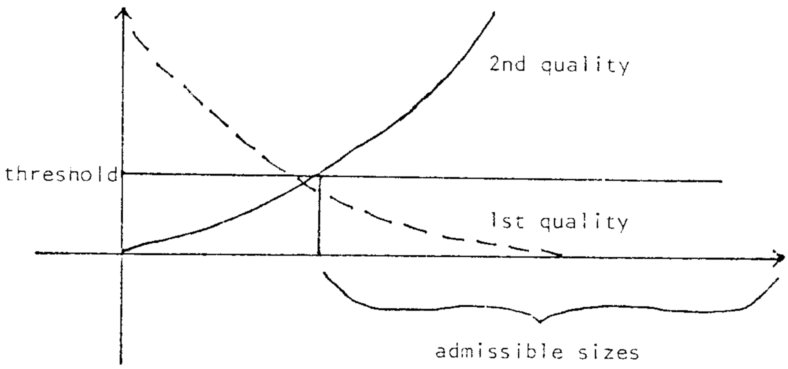

FIGURE 5b

The global admissible sizes are those belonging to the intersection of the two intervals of particular admissible sizes. When this intersection is non-empty, then the entity can be characterized as a good one; it can exist in the frame of the thresholds restrictions. In the opposite case a critical situation occurs: the two restrictions imposed by thresholds cannot be simultaneously observed. The entity cannot exist as a "good" one.

The contradiction principle implies the existence of at least one quality-indicator as in Figure 5a. A very important statement can be inferred on the basis of this figure: the interval of "good" sizes is every time a finite one. Therefore, there are not good sizes arbitrarily large. We have obtained again the trivial finding that everything has a limit.

> It is evident for anybody ready to think about what he observes that nothing in the world can grow without limits: Things growing stop to do so when reaching some determined size. This fact seems so evident, that most people — and most scientists — accept it without further thought and without posing the questions: at which size, why, and by what operations does an entity stop its growth. [Friedman, 1978; our emphasis]

A general corollary of the contradiction principle seems to be very important for today's world: any action intended to improve something has at least a negative consequence (eventually, among other positive ones). Let us think of economic growth, the demographic growth, or, following Mills (1978), "the school enrolment ratios, hospital beds per 1,000 of populations, percentage of households with access to safe drinking water." The last three indicators, positive at first sight, can correspond to a bad situation: "more educated unemployment, accentuated rural-urban migration, unutilized and badly staffed hospital facilities" (Mills, 1978).*

Annex

Let Σ having m individuals be structured on three levels with r subgroups on level 2, the i-th subgroup consisting of m(i) individuals. We have:

$$2 \leqslant r \leqslant m/2,$$

$$m \geqslant 3,$$

* Note: Very useful remarks by Professor Solomon Marcus and by Dr. Cătălin Mamali are acknowledged by the author.

$$2 \leqslant m(i) < m,$$

$$\sum_{i=1}^{r} m(i) = m.$$

The total effort of parcelling is

$$TEffort = kr^{\beta} + \sum_{i=1}^{r} km(i)^{\beta}$$

whereas the efforts of two-levels parcelling is:

$$Effort\ (m) = km^{\beta}.$$

Let us consider the difference:

$$A(\beta) = Effort\ (m) - TEffort = k[m^{\beta} - r^{\beta} - \sum_{i=1}^{r} m(i)^{\beta}].$$

We shall show that $A(\beta) > 0$ for any $k \geqslant 0$, $\beta \geqslant 2$ and thus we shall obtain that the parcelling in a three-level hierarchy has a lower cost than the parcellation in a two-level hierarchy.

Assertion 1. In the above conditions we have

$$m^2 > r^2 + \sum_{i=1}^{r} m(i)^2.$$

Indeed, it is sufficient to show that

$$m^2 > \sum_{i=1}^{r} [r + m(i)^2]$$

since $r^2 = \sum_{i=1}^{r} r$. Since $m = \sum_{i=1}^{r} m(i)$ we have $m^2 = \sum_{i=1}^{r} m \cdot m(i)$, hence it is sufficient to show that

$$\sum_{i=1}^{r} m\ m(i) > \sum_{i=1}^{r} (r + m(i)^2)$$

hence it is sufficient to prove that

$$m\ m(i) > r + m(i)^2$$

for each i and that the inequality is proper for at least one i.

As $m(i) \leqslant m - r + 1$ for each i we obtain $r + m(i)^2 = r + m(i)$. $m(i) \leqslant r + m(i)(m - r + 1) = m(i)\, m + [r + m(i) - r\, m(i)]$.

If $m(i) = 1$, then clearly, $m\, m(i) \geqslant r + m(i)^2$. If $m(i) \geqslant 2$, then $r\, m(i) \geqslant m(i) + r$ hence we obtain $r + m(i)^2 \leqslant m\, m(i)$. The inequality is proper when $m(i) > 2$ or $r > 2$. If $m(i) = r = 2$, then $m \geqslant 3$ and again $m\, m(i) > r + m(i)^2$. Assertion 1 is proved. For $\beta = 2$ we have obtained that $A(\beta)$ is strictly positive.

<u>Assertion 2</u>. Let $A'(\beta)$ be the derivative of $A(\beta)$. We have $A'(\beta) > A(\beta)$ for any $\beta > 2$ with $A(\beta) > 0$.

Indeed, $A'(\beta) = m^\beta \ln(m) - r^\beta \ln(r) - \sum_{i=1}^{r} m(i)^\beta \ln[m(i)] > m^\beta \ln(m) - r^\beta \ln(m) - \ln(m) \sum_{i=1}^{r} m(i)^\beta = \ln(m)\, A(\beta)$. As $m \geqslant 3$, $\ln(m) > 1$, hence $A'(\beta) > A(\beta)$ whenever $A(\beta) > 0$.

<u>Assertion 3</u>. $A(\beta)$ is strictly increasing on the interval $(2, \infty)$.

According to Assertion 2 it is sufficient to prove that $A(\beta)$ is strictly positive on $(2, \infty)$.

Let us suppose that there is $x_0 \varepsilon (2, \infty)$ such that $A(x_0) < 0$. As $A(\beta)$ is a continuous mapping, it has the Darboux property, hence there is at least a point x_1 such that $2 < x_1 < x_0$ and $A(x_1) = 0$. Let x_2 be the nearest to 2 point x_1 with the above properties. Consequently, for any x_3, $2 < x_3 < x_2$, we must have $A(x_3) > 0$. From Assertion 2 it follows that $A(\beta) > 0$ on the interval $(2, x_2)$, hence $A(\beta)$ is strictly increasing on this interval. For any $x_3 \varepsilon (2, x_2)$ we have $A(x_3) > A(2)$. As $A(\beta)$ is a continuous mapping, we obtain $A(x_2) > A(2)$. Contradiction. The existence of x_0 is impossible and the proof is complete.

Perhaps the results in this annex can be obtained for smaller values of β. However, for $\beta = 1$ the opposite case is true hence β must be, in some sense, significantly greater than one.

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III. ON THE COHESIVENESS INDICATOR OF A SOCIAL GROUP

Monica Tătăram

1. Introduction

In order to answer the question contained in the title of his paper, "What is a Quality of Life Indicator?"[1] Mario Bunge tries to give a formalization of the notion of a reliable indicator, in particular of a reliable social indicator. He also assesses the possibility of constructing a quantitative, hence an objective indicator, for any social aspect, even for those which are subjective. As an exemplification, he defines the social cohesiveness indicator by the following formula:

$$k(c) = |A \cap B| \cdot |A \Delta B|$$

where: C denotes a community split into two social groups: A and B; $|A \cap B|$ stands for the number of the elements belonging to both groups; while $|A \Delta B|$ stands for the number of the elements belonging to only one of the groups.

Starting from these premises, we propose another definition for the notion of an indicator and consequently of a reliable social indicator, and we improve the formula of the cohesiveness indicator. Furthermore, the formula is verified with respect to the new definition of the reliable social indicator.

It is at this point that the critical groupsize concept of Yona Friedman[5] was used in order to elucidate one of the limit situations, namely when every individual in C belongs to both groups A and B. In this case, we practically have a single group that we can investigate with respect to Friedman's considerations regarding the social structure of groups.[4] The critical groupsize is precisely a limit that, when not trespassed, ensures us that the cohesiveness of a social group will stay above zero.

2. The Notion of an Indicator, as Defined by Mario Bunge

Following Galtung, Poleszynski, and Wirak,[3] Bunge claims that

> there is no such thing as an indicator in itself; every indicator points to, or is a token of something else. In other words, an indicator is an observable trait of a thing that is rightly or wrongly assumed to point to the value of some other trait, usually an unobservable one, of either the same or a different thing.

As pointed out by these observations, many indicators are ambiguous: they indicate only one aspect of a thing or they are used to indicate several different things. One way to avoid ambiguity seems to be by the use of several indicators, checking one another, i.e., the use of vectors of indicators. Another way, much more highly recommended by Bunge, is to adopt a reasonably true theory interrelating the two variables: the indicator and the indicated thing.

> All I am claiming is that theory alone can justify the use of an indicator or its replacement by another, because only a theory interrelates variables and thus is in a position to explain some of them by others.

Furthermore, Bunge gives the following definition of the notion of an indicator:

Definition 1:[1] Let S be a set of variables (or rather property-representing functions) in some field of scientific research. Further, let I be a binary relation in S such that "Ixy" is interpretable as "variable x indicates variable y" and endowed with the following properties:

1. asymmetry: if Ixy, then $\neg$Iyx
2. transitivity: if Ixy and Iyz, then Ixz
3. Ixy only if: (a) x and y are either functionally related (i.e., there exists a function f such that y = f(x)) or statistically

correlated (i.e., $r_{xy} > 0$); (b) x is observable, countable or measurable without the help of any other variable in S.

Then, the set $S_y = \{x \text{ is in } S/Ixy\}$ is a set of indicators of y.

We may notice that:

1. The fact that the set S of variables is connected with a specified field of scientific research eliminates the possibility that an economic variable would indicate a social aspect. For instance, in spite of the Valaskakis and Martin[8] criticism of the GNP flaws, which are caused by its aggregated character (see Păun[7]), the GNP remains a strong and significant indicator from the economic field which points to the social standing of a person or a nation. Or, according to Definition 1, this is not correct.

2. By Definition 1 only a set of indicators of a given variable is obtained and not the set of its indicators. We presume the reason to be the following: "Whether or not a given variable indicates (points to) the values of another variable is not a matter of convention but of hypothesis, i.e., corrigible proposition." So, the set S_y of indicators of the variable y can be improved. A wider discussion of this point will follow in section 3, based on Calude, Marcus, Păun[2] and on Mallmann, Marcus.[6]

3. The definition contains two ambiguities. The first one concerns the fact that in order to define a set of indicators of a given variable, a binary relation is used, that is interpreted as ". . . is an indicator of . . .," endowed with four properties. The first two, asymmetry and transitivity, make it resemble the order relation in R; the last two cannot be mathematically formulated. So, we cannot avoid this ambiguity by first defining the binary relation I in S and then the indicator as a variable x of S placed in the relation I with a given variable y of S. The only thing we can do is to regard the statement "variable x indicates variable y" with respect to Bunge's considerations[1] quoted above, i.e., "variable x points to variable y."

As for the second ambiguity: according to the assumption (3b) a variable x of S has to be "observable, countable or measurable without the help of any other variable in S," in order to be an indicator. So, if the variable x is an indicator of the variable y then y cannot indicate a variable z, as y is no more observable without the help o any other variable in S. So the transitivity of the binary relation I (assumption 2) cannot hold.

Let us take the following example: let S be the set of the socio-economic indicators of our economy. They are contained in the economic plans for development and, in the financial form, in the budgetary classification. We have the following situation: "the number of pupils in high schools" (let us denote it by x) is an indicator for "high-school expenses" (let us denote it by y) which in turn, is an indicator for "training expenses" (let us denote it by z). It is obvious that so far we were on the hypothesis of Definition 1. But while the variable x, "the number of pupils in high schools," is clearly measurable without the help of any other variable in S (i.e., without the help of any other socio-economic indicator) y is not. Indeed, in order to obtain y, "high school expenses," we have to consider not only "the number of pupils in high schools," i.e., the variable x, but also, for instance, "handbooks and school supplies" or "household expenses" or "equipment expenses," which are all socio-economic indicators. So, y is measurable, but only with the help of some other variables in S. Nevertheless, y is in the relation I with z and they are functionally related (the variable y is a term in the sum that is variable z). So, is y, "high school expenses," an indicator for z, "training expenses"?

Following the budgetary classification, the answer is "yes." Following Bunge's definition, the answer is "no." This situation proves our point and entitles us to make an attempt to improve Bunge's definition.

3. A New Definition of the Indicator, without the Transitivity Assumption

Let us first give up the transitivity assumption in Definition 1. It becomes:

Definition 1'. Let S be the set of variables in any scientific research field. Let S_0 be the set of variables which are directly observable, countable, or measurable (i.e., without the help of any other variable in S). Obviously $S_0 \subset S$. Let $\bar{S}$ be the set $S \setminus S_0$ (i.e., the complement of S_0 in S). Further, let I be a binary relation in S such that "Ixy" is interpretable as "variable x indicates, points to, variable y," with the following property:

"Ixy" only if: (a) x and y are either functionally related or statistically correlated; (b) x is in S_0 and y is in $\bar{S}$. Then the set

$$S_y = \{x \text{ is in } S_0 \mid Ixy\}$$

is the set of the indicators of y in $\bar{S}$.

According to this definition, the binary relation I is obviously asymmetric and non-transitive. The sets S_y, y in $\bar{S}$, are subsets of S_0, and, for some y, z in $\bar{S}$, the intersection $S_y \cap S_z$ may be non-empty, while the sets $\bar{S}_x$, x in S_0, defined as:

$$\bar{S}_x = \{y \text{ is in } \bar{S} \mid Ixy\}$$

are subsets of $\bar{S}$ and, for some x, t in S_0, the intersection $\bar{S}_x \cap \bar{S}_t$ may be non-empty. So, by the help of the binary relation I we may split the set S_0 into several non-disjoint sets S_y, y in $\bar{S}$, containing the variables which indicate the variable y; or the set $\bar{S}$ into several non-disjoint sets $\bar{S}_x$ containing the variables which are indicated by the variable x, x in S_0.

Now, we shall unify and formalize these considerations, along with the second observation concerning Definition 1, on the basis of Calude, Marcus, Paun[2] and of Mallmann, Marcus.[6]

Let: $\quad i: S_0 \to 2^{\bar{S}}$

($2^{\bar{S}}$ denotes the set of subsets of $\bar{S}$), be a mapping which associates to each directly observable variable x in S_0, a set of variables y in $\bar{S}$ such that x indicates y. A variable x in S_0 such that i(x) is empty is called parasitic. A variable y in $\bar{S}$ is parasitic provided that y does not belong to $i(S_0)$. We suppose that the usefulness axiom (no variable x in S_0 is parasitic; i.e., for every variable y in $\bar{S}$, there exists at least one variable x in S_0 such that x indicates y) are both fulfilled, though the second axiom introduces a strong restriction in $\bar{S}$ and consequently in S.

Let us take an example, in fact let us come back to the set S of the socio-economic indicators. There exists a set of indicators called substantiation and calculation indicators of the financial indicators (for instance "the number of pupils in high schools" is precisely one of the substantiation and calculation indicators for the financial indicator that is "high-school expenses"). So we may consider that the set of the substantiation and calculation indicators is the set S_0 in Definition 1' and that the set of the financial indicators contained in the budgetary classification and in the economic plans is the set $\bar{S}$ in Definition 1'. Then, by the help of the function i we can associate to each substantiation and calculation indicator x in S_0 the financial indicators y in $\bar{S}$ in which calculation x occurs itself. Then we may see that the number of the parasitic financial indicators is very small (2) such that, at least in this example, we may assume that the completeness axiom is fulfilled.

Further, a partial order relation is introduced in (2) that enables the authors to say that one state of illness is more negative than another. Unfortunately such an ordering cannot be introduced in $\bar{S}$, as we cannot always compare two indicated variables y and z in $\bar{S}$.

Now we shall rewrite the definitions given in (2), with respect to the sets S_0, $\bar{S}$ of variables considered here.

We shall say that <u>an indicator x in S_0 is dependent with respect to an indicator u in S_0</u> if any variable that x indicates is also indicated by u (i.e., if $i(x) \subset i(u)$).

We shall say that an indicator x in S_0 is dependent with respect to a set S_0^* in S_0 if any variable that x indicates is also indicated by a suitable set of indicators in S_0^* (i.e., if $i(x) \subset i(S_0)^*$).

A set S_0^* of indicators is said to be <u>independent</u> if no indicator x in S_0^* is dependent with respect to $S_0^* - \{x\}$.

A set S_0^* of indicators is said to be <u>complete</u> if any indicator which is not in S_0^* is dependent with respect to S_0^*.

A set S_0^* of indicators is a <u>kernel</u> of S_0 if it is both independent and complete.

In our example the only kernel of S_0 is S_0 itself.

The characterization given to kernels in (2): a kernel is both a minimal complete set and a maximal independent set, is very important for our example. Indeed, if one tries to improve the set of the substantiation and calculation indicators for the financial indicators, one has only to see if by adding a new indicator to S_0 this one is not dependent with respect to S_0 or if by eliminating an indicator of S_0 there exists a financial indicator that can no longer be calculated.

Also of a great importance is another function constructed in C. Mallmann, S. Marcus's paper,[6] and that we shall rewrite as follows:

$$h: \bar{S} \to 2^{S_0}$$

which associates to each variable y in $\bar{S}$ the set of the variables x in S_0 that indicate it.

In our example, h would associate to each financial indicator its substantiation and calculation indicators. A very immediate reason for the affirmation regarding the importance of h, is its help in the search to find a suitable codification for both the financial indicators and the substantiation and calculation indicators, namely a codification able to eliminate the ambiguities and to solve the difficulty arising from the fact that some calculation indicators substantiate several different financial indicators.

4. Another Definition of the Indicator; Rewriting other Bunge Definitions

Let us reformulate the assumption (3b); Definition 1 becomes:

Definition 1''. Let S be the set of variables in any scientific research field and I a binary relation in S, such that "Ixy" is interpretable as "variable x indicates (points to) variable y" and endowed with the following properties:

1. asymmetry: if Ixy then ¬Iyx
2. transitivity: if Ixy and Iyz then Ixz
3. Ixy only if x and y are either functionally related or statistically correlated
4. x is observable, countable, or measurable.

Then the set S_y = {x is in S / Ixy} is a set of indicators of y.

We shall keep this latter definition and we shall reformulate the definitions 2, 3, and 4, given by Bunge[1] in this respect, also avoiding certain ambiguous formulations that they contain.

Definition 2'. Let S_y be a set of indicators of variable y.

Then, if x is in S_y (in the sense of Definition 1''), x is a reliable indicator if the relation between x and y, (1) belongs to some theoretical model; and (2) has been well-confirmed by empirical tests.

Again, we may find the basis of this definition in the following considerations:[1]

> . . . only a theory explaining the mechanism whereby unobservable X is manifested as observables Y_1, Y_2, . . ., Y_n, is

capable of justifying the choice of the latter set of inidcators rather than any other observable traits, to estimate the value of X. . . . And for an indicator-indicated relation to be reliable, it must have passed certain empirical tests.

Definition 3'. Let variable x be an indicator of variable y in the sense of Definition 1'' (according to Bunge's definition, x must be a function representing properties of some social system or subsystem). If y is a social variable, then we may say that x is a social indic r for y.

Definition 4'. Let variable x be a social indicator for variable y (in the sense of Definition 3'). Then, x is a reliable social indicator if the relation between x and y, (1) belongs to some theoretical model; and (2) has been well-confirmed by empirical tests.

5. The Social Cohesiveness Indicator as Defined by Bunge: A New Formula for It

Now, we may recall the conditions a variable x has to accomplish in order to be a reliable social indicator for a variable y:

a. The variable x and the variable y must belong to the set S of the variables in any scientific research field (see Definition 1''). Furthermore, the variable y must be a social variable and the variable x must be a function representing properties of the social system of the subsystem to which y belongs (see Definition 3').
b. The variable x and the variable y must be in the binary relation I defined in S and interpretable as "variable x indicates (points to) variable y." According to Definition 1'', this relation is irreflexive, asymmetric, and transitive.
c. According to the same definition, the variable x must be observable, countable, or measurable.
d. The variable x and the variable y must belong to some theoretical model (see Definition 4').
e. The relation I between variable x and variable y has been well-confirmed by empirical tests.

Unfortunately, the social cohesiveness indicator defined by Bunge in his paper[1] does not satisfy these conditions. In fact, it satisfies the first three (and mainly the third, as variable x is countable: we have only to enumerate the number of people belonging to the intersection $A \cap B$ and respectively to the symmetric difference $A \Delta B$) but it does not satisfy the most important requirements for being a reliable social indicator, i.e., the conditions (d) and (e). Here are the reasons.

1. The cohesiveness indicator is defined with the help of an arithmetical operation between two other social indicators: the degree of differentiation of C, denoted by d(C) and defined as:

$$d(C) = |A \Delta B|$$

and the degree of participation of C, denoted by p(C) and defined as:

$$p(C) = |A \cap B|$$

The social cohesiveness indicator, k(C), defined by Bunge as:

$$k(C) = p(C).d(C)$$

cannot be correct as the social cohesiveness of a community cannot equally improve by the growth of the degree of participation as well as by the growth of the degree of differentation. So, we propose the following formula for k(C):

$$k(C) = \frac{p(C)}{d(C)} = \frac{|A \cap B|}{|A \Delta B|}$$

which shows that, the greater the degree of participation, the greater the cohesiveness is, and that the smaller the degree of differentiation, the greater the cohesiveness.

This new formula matches the results of a sociological investigation performed in 1976 by the Laboratory for Sociological Studies and Research IPCT[9] on a population with 253 members formed by 61 workers who owned their apartments, 61 intellectuals who also owned their apartments, 68 workers who rent their apartments and 63 intellectuals who rent their apartments. The answers obtained led to the identification of the comparative characteristics of the apartments which were private property and of those which were rented, as well as to the identification of the way in which owners and renters perceived and evaluated their dwellings. The results of the test, interpreted in the terms of our formulas and definitions, showed that social cohesiveness increases with social participation and decreases with social differentiation.

2. Bunge claims that: "In order for C to be cohesive, it must be partitioned into at least two groups that somehow complement one another."

That means that, for Bunge, the social cohesiveness indicator k(C) is maximum when one of the groups of the community C is the complement of the other. We shall prove that his statement cannot hold.

Let us suppose that the community C is split into two groups A and B which are disjoint. In terms of the theory of sets this means that:

$$A \cap B = \phi$$

(where ϕ denotes the empty set), and that

$$A \triangle B = (A \cup B) \setminus (A \cap B) = A \cup B \setminus \phi = C \setminus \phi = C$$

while in terms of our definition, this means that the degree of participation is zero:

$$p(C) = |A \cap B| = |\phi| = 0$$

and the degree of differentation is maximum

$$d(C) = |A \triangle B| = |C|$$

Consequently the social cohesiveness indicator is minimum

$$k(C) = \frac{p(C)}{d(C)} = \frac{0}{|C|} = 0$$

namely: when the community C is split into two disjoint groups A and B, the cohesiveness indicator of C is zero. And it is zero in both cases: when considering Bunge's formula:

$$k(C) = p(C).d(C) = 0 . |C| = 0$$

as well as when considering the proposed formula

$$k(C) = \frac{p(C)}{d(C)} = \frac{0}{|C|} = 0$$

So the statement of Bunge is contradicted by the very formula that is used in order to calculate the cohesiveness of C. Our demonstration is well confirmed by the results of the investigation cited above: the owners have more personal goods in their apartments while the renters were pleased with what was provided; the owners have taken better care of their apartments than the renters.

Furthermore, Bunge claims that "total participation erases all differences; i.e., corresponds to the case in which C = A = B. And this extreme situation is not conductive to cohesiveness but to competition."

Let us investigate this other limit situation when the community C is practically formed by a single group. This means that

$$A \cap B \simeq A \simeq C$$

and

$$A \triangle B = (A \cup B) \setminus (A \cap B) = C \setminus C = \phi$$

and consequently

$$p(C) = |A \cap B| \simeq |C|$$

$$d(C) = |A \triangle B| = |\phi| = 0$$

that leads to a nondetermination:

$$k(C) = \frac{p(C)}{d(C)} = \frac{|C|}{0}$$

which forbids us to decide, as we did before, whether we have total or zero cohesiveness.

If we consider this case (when the community C is formed by a single group, A, for instance) as a limit of the case when the community C is formed by a group A and a part of it, B (in terms of the theory of sets: B is included in A, is a subset of the set A), we can assume that this is the case when social cohesiveness is maximum.

Indeed, if the community C is such that the group B is included in the group A, then the degree of participation will be:

$$p(C) = |A \cap B| \simeq |B|$$

and the degree of differentiation will be:

$$d(C) = |A \triangle B| = |(A \cup B) \setminus (A \cap B)| = |A \setminus B| = |C_C B|$$

(where $C_C B$ denotes the complement of the set B in C) and consequently the social cohesiveness indicator will be

$$k(C) = \frac{|B|}{|C_C B|}$$

With the help of this formula, we can see that the greater the social subgroup B of the group A (i.e., C) is (and consequently the smaller its complement is) the greater the social cohesiveness will be. In fact, when the subgroup B differs from the group A (i.e., C) by only a single individual (i.e., the number of B will be

$$|B| = |C| - 1$$

and the number of $C_C B$ will be

$$|C_C B| = 1)$$

then the value of the social cohesiveness indicator will be maximum as it will be equal to $|C| - 1$. Indeed:

$$k(C) = \frac{|B|}{|C_C B|} = \frac{|C| - 1}{1} = |C| - 1$$

It is only natural to assume that when the social subgroup B of the group A will be equal to A (i.e., the community C will be formed by a single social group) the cohesiveness of C will be maximum (common sense tells us so; the above mathematical tools are coherent with this statement). Nevertheless, Bunge claims this is a case of competition, and the formula of the social cohesiveness indicator, interpreted from the mathematical point of view, holds a non-determination.

This is the reason for the use of Yona Friedman's considerations "About Critical Groupsize."[5] Although in this article Friedman does not explicitly use the term "social cohesiveness," his conclusions are useful in regard to the alternative that a social group has when trying to stay whole or to keep its structure.

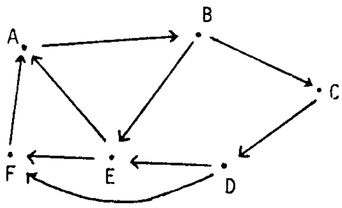
6. Yona Friedman's Concept of Critical Groupsize

We shall begin by presenting the frame of Friedman's conclusions.

Observing any human group, we see some exchange of signs: a communication process between members of the group gets established . . . after a certain exchange of signs (or words) some members of the group behave differently. We can thus conclude that these individuals were influenced by communicating with the others.[6]

Friedman used a diagraph in order to show these influences, taking the members of the group for the points of the diagram and their influences as the oriented lines of it.

Here is an example:



This enables him to observe that:

Those members of the group who exert many influences are necessary important members, and those ones who receive many influences and exert but few are members of less importance . . . influences are not necessarily exerted directly. Many of them are transmitted by intermediaries. . . . We can suppose that an influence transmitted by an intermediary loses part of its "credibility." We can thus conclude that a transmitted influence is weaker than a direct one.

In order to point out this loss of influence, Friedman assumes that the first intermediary transmits only one half of the influence received, the second one only one third, the third only one quarter and so on. In this way, by summing up the influences exerted and the influences received by each member and then by calculating the difference between the total influences exerted and the total influences received (he calls this difference "the balance of influences of the individual") Friedman obtains a hierarchy of the members of the group.

Resuming our example, we get:

from \ to	A	B	C	D	E	F	Total influences exerted
A	0	1	1/2	1/3	1/2	1/3	2 2/3
B	1/2	0	1	1/2	1	1/2	3 1/2
C	1/3	1/4	0	1	1/2	1/2	2 7/12
D	1/2	1/3	1/4	0	1	1	3 1/12
E	1	1/2	1/3	1/4	0	1	3 1/12
F	1	1/2	1/3	1/4	1/3	0	2 5/12
Total influences received	3 1/3	2 7/12	2 5/12	2 1/3	3 1/3	3 1/3	

The balances of influences will be:

B	1 1/2
D	9/12
C	2/12
E	3/12
A	8/12
F	11/12

Friedman defines the "social structure" of the group as a function of its "mathematical structure" outlined by the balance of influences of each member of the group:

> If the balances of influences of all members are roughly the same, we have an "egalitarian group"; if the balances of influences of the members are regularly decreasing (as in the example above) we have a "hierarchized group"; if the balances of the members are distributed on two levels we have an "oligarchical group."

Further on, Friedman introduces two parameters that help to characterize humans. These are "valency" and "transmission capacity."

> We call "valency" the largest number of influences which can be assimilated by a human during a "given reference time" (i.e., immediately, hourly, yearly, etc.).
>
> "Transmission capacity" indicates a measure of human error in transmitting an influence received from another individual and addressed to a third one. This measure of error can be expressed by the largest number of transmissions which do not alter the original message by making it useless.

Both these "biological characteristics" of human groups are observable quantities which can be determined empirically. Also, they are dependent on the context. The first one depends upon the assumed reference duration, the second depends upon the chosen code of language. Valency as well as the transmission capacity have effect on the "decision rapidity within the group."

Now, we may state the conclusions of Friedman.

1. "A human group, characterized by some 'social structure' cannot work except if the number of its members stays under a threshold number. . . . We will call this threshold 'critical groupsize'."

2. The formula for the "critical groupsize" would depend upon:

a. the social structure of the group (which is a function of the mathematical structure of the group, here defined and calculated with the help of the balances of influences of the numbers of the group);

b. the specific human "valency" (which is a function of the reference duration, here determined empirically with the help of experiments and given as a table);

c. the specific human "transmission" or "channel capacity" (which is a function of the chosen code of communication, here determined empirically with the help of experiments and also given as a table).

So the formula for the critical groupsize would be:

$$G: \; R[s(m), \smile \text{table } v(t), \smile \text{table } c(1) \smile]$$

where: G denotes the critical groupsize, s denotes the social structure, m denotes the mathematical structure, v denotes the valency, t denotes the reference duration or reaction speed, c denotes the channel capacity or transmission capacity, 1 denotes the language or code.

3. A human group larger than the critical groupsize corresponding to its social structure has the following alternatives:

a. it can split into groups which will conserve the original social structure;

b. it can stay a whole but it must change its social structure.

Indeed, the social structure of a group can be expressed with the help of the following formula:

$$s(m): \; R'' \text{ (Table G, Table } v(t), \text{ Table } c(l) \text{)}$$

i.e., with the help of three sets of variables each determined with the help of experiments (the critical groupsize included) and given as tables.

7. Cohesiveness and Critical Groupsize

Now we have the necessary tools to investigate the limit case of the social cohesiveness indicator formula of Bunge where his conclusions were somehow ambiguous. Friedman offers us a very simple solution, provided the number of the members of the community C stays under the critical groupsize. For instance, if the community C is formed by a single group organized like an egalitarian group (see Friedman's definition[5]) and if its "valency" is 6 and its "channel capacity" is 5, then the community C will stay a whole and will keep its egalitarian structure (in other words its cohesiveness will be maximum) provided the numbers of C do not exceed 16, i.e., the critical groupsize calculated by Friedman for egalitarian groups with valency 6 and channel capacity 5.

In the other case, when the numbers of the community C exceed the critical groupsize corresponding to its social structure, the social cohesiveness of C will not be maximum any longer. Indeed, in this case the community will have either to split into at least two groups (and if those groups are disjoint then k(C) will be zero) or to change its social structure.

The question that arises now is the following: What are the effects of the change of the social structure of a group on its cohesiveness? Without trying to answer this question, let us say that Friedman's conclusions plead for maximum cohesiveness when the community C is formed by a single group and contradict Bunge's assumption of total competition in this case.

8. Conclusions

As Bunge points out in his paper, "unfortunately, there are no rules for inventing anything, in particular sensitive social indicators. The most one can do is to propose and to discuss examples." It is precisely what we tried to do here: starting with Bunge's formula for the social cohesiveness indicator (a quantitative indicator for social cohesiveness which is not directly observable) we tried to verify the definition, i.e., the correctness of the formula as we could not have the means to put it to tests, even empirical ones. In this attempt we came across a situation where a decision regarding the values of k(C) could not be taken. Friedman's concept of critical groupsize was of some help but could not elucidate the matter completely. So we must agree with Bunge that "we need more theoretical (if possible mathematical) models and more methodological studies to deal with the subject."

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IV. THE RELEVANCE OF THE "ELECTRE" METHOD IN STUDYING THE QUALITY OF LIFE

Titus Priboi

This paper examines the conditions under which the quality of life may be measured by using a method of multidimensional analysis of data, namely the Electre method. In this context we try to get a clearer understanding of the quality-of-life concept, thus helping future research in the field.

The paper refers to the experimental use of the Electre method applied to a small number of social indicators, concerning some variants of the concept of quality of life.

Four aspects of the concept of quality of life are essential for our presentation:

- its pluri-dimensional character
- the heteroclite character of its dimensions
- the dependence of its definition on value systems
- polysemy

These aspects are not independent; we only separate them for an easier approach. The same criterion was used when seriating them.

First, we shall briefly analyse some consequences of the above-mentioned aspects on the methods of measuring, structuring, etc., the concept of quality of life. Then, we shall resume some of the problems, referring to a case study.

Among pluri-dimensional concepts, the concept under consideration — quality of life — is characterized by its potentially great number of dimensions and the even greater number of the indicators which define it.

For measuring the concept, organizing indicators which define it, comparing levels of the quality of life, it is advisable to resort to devices used for building indices (aggregate measures) or to methods of multidimensional analysis of data. To make our presentation coherent we have to recall that these devices and methods depend on the shares of data and indicators, which, sometimes, lead to subjective valuations, to value systems.

To this we should add the heteroclite character of the dimensions and indicators to which we refer the concept. Some of them are related to physical or material dimensions, others to social or human relations, states, and valuations of subjective states. This characteristic recalls previous debates on the possibility of a unique standard, to which data of different kinds may be referred. Such a standard is, in economy, the monetary value. It is the means of synthesizing in only one number and of comparing data which are very different in other respects.

There is no such possibility when dealing with the concept of quality of life. However, we may aggregate and compare the data by using the already-mentioned methods.

Third, the dimensions and data depend on subjective valuation. So, we come to define the set of data and indicators which will represent the concept of quality of life, and to identify the nature of these data. In such a context the distinction between indicators based on objective elements and those resulting from subjective valuations is obvious.

Even if we eliminated subjective indicators we could hardly fail to refer indicators to certain value systems, if the methods of dealing with data introduce coefficients which are important for weighing the objective indicators.

Two possibilities are to be considered: (1) It will be easier to agree on the shares associated to the objective indicators, that is on a homogeneous/unitary, non-contradictory value system, subjacent to them; (2) the share system will be objectively established or will result from scientific investigations. Admitting that the latter possibility is theoretically feasible, there is still the problem of not over-simplifying the concept by the initial selection of data.

Considering subjective indicators, we cannot avoid the reference to value systems.

Last but not least comes the polysemy of the concept. Order in this respect may be reached by registering the main definitions of the concept of quality of life. This is one of the themes of Onicescu.[10]

In considering the polysemy of the concept, two basic strategies may be adopted:

- using various devices, we introduce a unique definition of the concept and, correspondingly, a unique set of indicators;
- the opposite alternative, that is, we use more concepts of life quality.

Each alternative may be segmented into more variants. For the former alternative, for example, a variant is suggested — although in a quite different context — by "Toward a Social Report." It proposes that social indicators should be sought only for problems generally agreed upon considering the concepts of progress and regress. The realism, or rather the accessibility of such an approach, is obvious. This does not mean, however, that it may avoid subsequent contestation. So, it seems reasonable to direct our efforts towards the choice of such indicators as could eliminate controversy.

The latter alternative may be illustrated by B. Cazès'[5] hypotheses. The author admits the possibility of building types of the quality of life, with different dosing of their components. Such an idea was suggested by the works of the anthropologist Ruth Benedict, who studied some Indian ethnic groups in North America, using what she called the "Apollo type" and the "Dionysian type." Cazès considers that such an approach avoids normative judgements, according to which we could decide which type is superior to the other.

This short presentation is just the general conceptual framework for our study of a problem analogous — on a small scale — to the problems connected with the concept of quality of life.

Multidimensional Concepts and Methods of Multidimensional Analysis of Data; Electre

It is more or less natural to relate the multidimensional concepts of social sciences to the methods of multidimensional analysis of data.

In Bernard and Besson[1] and Bertier and Bourouche[2] such methods are indicated as: the hierarchical analysis, the voluntarist method, the rule of the simple majority, successive permutations, Electre partitions, binary relations, preordering of k classes, median relations between k elements, "Iphigenie." We may even extent this list by adding the methods suggested by Onicescu[10] (the so called method of the dominant, the method of ordering according to characteristics).

For reasons which will be mentioned further on, we are interested in the Electre method; parts of it, which are connected with our paper, will be presented here.

Let E be a finite set of m objects: $E = \{e_1, e_2, \ldots, e_m\}$. Each object may be looked at and characterized from n points of view, or according to n criteria, which we shall denote by $x_1, x_2, \ldots, x_n$. Generally, these points of view may have different meanings, such as:

- presence or absence of a certain quality;
- recognition of a characteristic or valuation of a qualitative element;
- the degree of reaching a goal;
- the results of an economic calculus, etc.

For example, a factory, before starting to make a new series of products, has to analyse them from the following points of view of commercialization, assimilation, etc. Since the objects are very different (chemical compounds, prehistoric objects, animals, conditions for a competition) the points of view are different too. Analysing an element e_i from a point of view x_j we get a result a_{ij}, which may be a number, a mark, or a qualitative valuation of the object, as those mentioned before.

The method may be used if any two elements of E can be compared from the point of view of each criterion.

We associate to any element e_i, qualitatively valued, an appreciation which may be scaled. It has been pointed out that we should assign to the results of these appreciations (a_{ij}) the meaning of utilities estimated by using any method.

Let us denote by $\bar{x}_j$ the set of results obtained when analysing, characterizing the elements of E, from the point of view x_j, and let us call "state" an element of $\bar{x}_j$. So, looking at e_i from the point of view x_j, we shall get its state, which will be an element of the set of states $\bar{x}_j$.

Here are a few examples of such sets of states:

{0, 1}

{yes, no, I don't know}

{bad, medium, good, very good}

{etc.}

Since an element e_i from E is considered from n points of view, we associate to e_i either n states from $\bar{x}_1$, $\bar{x}_2$, . . ., x_n, or its multi-dimensional state, which is an element of the cartesian product

$$x = \bar{x}_1 \times \bar{x}_2 \times . . . \times \bar{x}_m$$

We may organize the elements (objects), criteria and results (obtained by characterizing the elements according to the criteria) as follows:

TABLE I

	x_1	x_2		x_n
e_1	a_{11}	a_{12}	. . .	a_{1n}
e_2	a_{21}	a_{22}	. . .	a_{2n}
⋮	⋮			
e_m	a_{m1}	a_{m2}	. . .	a_{mn}

Since the n points of view or criteria are not equally important, we use a system of weighting coefficients: $\{P_1, P_2, \ldots, P_n\}$ for weighting the criteria. Different devices may be used for establishing the weighting system, since the considered method does not suggest a certain unique device.

What are the results?

If E is the set of the considered objects then Electre I allows us to select a subset of E, consisting of the most interesting objects, selected according to all n points of view, and thus it introduces a dichotomy between objects to be kept and those to be rejected. Electre II will introduce a total preorder, that is an order accepting <u>ex aequo</u> cases.

For example, if E are these possible action alternatives of a decision-maker and $x_1, x_2, \ldots, x_n$ its decision taking criteria, Electre allows us to order the alternatives according to all the considered criteria — that is, according to complex criteria — and to choose the best alternative (considering both positive and negative aspects of various alternatives).

Some Comments

The method has been applied to various economic problems or — as pointed out in Bertier and Bourouche[2] — it has been used — by the World Health

Organization — in selecting certain health assistance projects for various countries.

The method may be also used in dealing with certain social problems; we have employed it in a case study which raises questions similar to those raised by the quality of life.

In this context we have to recall that P. Vergès, when dealing with certain problems connected to social indicators, also makes some remarks concerning the factorial analysis and the Electre method. Speaking about Electre, he considers it necessary to examine what is hidden behind the pair "points of view/information," since indicators are, at a certain level, nothing else but information. At the same time, it is necessary to distinguish between "concepts/social indicators" on one hand, and "social problems and the information system" on the other hand. Electre considers only the latter relation, trying to organize information, resulting as an expression of various points of view, and looking for no conclusion regarding the relation between "concept" and "its social indicators." It is precisely here that both the correctness and the limits of the method, from the viewpoint of the sociologist intending to use it, are obvious. However, Vergès did not use the method in sociological studies — we know no application of it in that field. That is why he did not point out other problems raised by the use of the method.

The above-mentioned distinction is natural. It shows that Electre is a method of structuring, ordering data; it is independent of sociological concepts and suggests no theory concerning the relation between these concepts and their indicators. Applying the method to concepts of the social sciences, we cannot leave aside the relations which the method itself disregards. So, we have to consider: (a) the circumstances under which the method may be used; (b) the problems connected to the interpretation of the results in terms of the social sciences.

Despite all its inherent limitations, the method is still of interest to the specialist in the social sciences, who is so often facing problems connected to multidimensional concepts.

A Case Study; Theoretical and Methodological Remarks

The conception according to which we applied the Electre method to the case study has been presented by the author of this paper.[16] We are going to repeat here some ideas from that study. We intend to apply the Electre method to the data collected by means of questionnaires. Let us see what happens when replacing the general conditions with respect to which the method has been worked out by the conditions which the data questionnaires offer.

Some similitudes and oppositions are to be noticed. A general similitude is based on the following ascertained fact: since a completed sociological questionnaire contains information about the characteristics which the researcher is interested in, it generally follows that, using the questionnaire, we may associate to an individual a set of ordered elements $N = \{g_1, g_2, . . ., g_n\}$, where g_i are the indicators with respect to which the selected individuals are characterized. The elements g_i are numbers, order numbers, marks replacing qualitative attributes, etc. Such a set defines the state of the individual at the moment t, or rather his multidimensional state, defined simultaneously with respect to more points of view or criteria. The point of view corresponds to the indicators by which the individual is characterized.

Once the individuals have been defined, we may organize them, according to their multidimensional states, in rather homogeneous groups, which in turn may be compared, ordered, considering them simultaneously from the respective points of view = indicators. This is one of the problems to which Electre may be applied.

Further similitudes and differences may be identified.

Here is such a difference: Electre used a unique system of weighing coefficients; however, the questioned individuals may value these coefficients differently. So, it is necessary to use a device which will enable us to establish a unique set of weighing coefficients.

Before proceeding further, we will consider Table 2 (which is partly analogous to Table 1).

TABLE 2

		x_1	x_2		x_n
P_1	e_{P_1}	s_{11}	s_{12}	. . .	s_{1m}
P_2	e_{P_2}	s_{21}	s_{22}		s_{2n}
⋮	⋮				
P_m	e_{P_m}	s_{m1}	s_{m2}		s_{mn}

Where: $P_1, P_2, \ldots, P_m$ stand for the m individuals of the sample, e_{P_i} for the concrete, complex state of the individual P_i, before being analysed, but which is supposed to permit characterization from n points of view (indicators); by $x_1, x_2, \ldots, x_n$ we have denoted the indicators (points of view) with respect to which each individual is characterized, and by s_{ij} the characterization of the individual P_i from the point of view of x_j. S_i is the multidimensional state of P_i, defined by the set of elements corresponding to P_i in the table.

Under the above-mentioned circumstances we notice that:

- we may relate the m objects in E to the concrete, complex states e_{P_i} $(1 \leqslant i \leqslant m)$ of the m individuals in the sample;
- if an object $e_i \in E$ is characterized from n points of view, the concrete state e_{P_i} of the individual P_i is also characterized from the point of view of the n chosen indicators: $x_1, x_2, \ldots, x_n$. The indicator x_i may be, for example, a satisfying indicator with respect to the working place.

Some asymmetries are also obvious. The method accepts a unique centre which estimates, considering m points of view. Since some of these estimates may be value-appreciations, translated qualitatively into such terms as good/bad, it follows that the object is evaluated with respect to a certain standard or norm. We shall distinguish between point of view and its subjacent norm; that means a point of view may be based on two or more different norms. Obviously if the norms vary, the estimates of the same object, from the same point of view (its state) will differ. In our acceptation, point of view is an abstract concept. We are bound to take this way since the estimations in the questionnaires are referred to different norms of different individuals.

Electre asks that one and the same point of view (should) be used — irrespective which one — in characterizing all the objects and that each point of view depend on one norm only. However, we do not find such information in the questionnaires. If the concrete states e_{P_i} are analogous to the objects e_i in E, then we notice that: (1) in a questionnaire we ask each individual P_i to characterize only his concrete, particular state s_{P_i} from n points of view, not the states of the other individuals; (2) the characterizations of P_i depend on his norms, which may differ from the norms of P_j.

We denote by S_j the multidimensional state of the individual P_j and by s_{ji} a component of S_j, representing the value, or indicator obtained when the individual P_j is characterizing his state from the point of view x_i. P_j is asked to refer to an aspect of his particular state, while another individual P_k, from the same point of view x_i, will refer to an aspect of his state S_k, making eventual use of another norm. The questionnaire does not ask P_j to characterize from the point of view x_i, the state s_{ki} of the individual P_k.

Under such circumstances, what will it mean to organize the individuals in groups and to order these groups by using Electre II?

The answer is not a simple one. It partly depends on the distribution of the individuals with respect to the characteristic we have called norm. Let us suppose that we have identified five ordered groups by using Electre II. The fifth group will consist of the individuals who have the best situation from the m points of view considered. If belongingness to this group depends only on norms, then the diagnosis of group 5 could be formulated as follows: "group 5 consists of the individuals having the best state, the state of each individual being referred to his own norm." However, the establishing and ordering of the groups also depend on the system of weighing coefficients. Thus the problem is actually more complicated.

Things change if the sample is homogeneous, that is the individuals use roughly the same norm. That means that the data base should be extended; sometimes we even come to the conclusion that two samples, at least, are needed.

There is still another alternative, which essentially consists in introducing standard devices for univocally characterizing the states dependent on subjective estimates. This standardization may be based on the results of scientific research. Generally, norms are introduced by an "arbitrary" decision, that is a decision fit to the given problem, such that, considering the norm and other information, we may univocally encode the states of the individuals. Although arbitrary — subject to valuations — the norm is explicitly formulated, thus offering a base for interpreting results. The above mentioned condition once satisfied, the data are no longer "a black box," but a "white" one, considering their meaning. In our case study we have used this method.

An Experimental Application of the Electre Method

Our main aim in the previously mentioned paper[16] has been to study not the quality of life but some problems of urban sociology, and that's why we have used only the variables specified below. Even under such circumstances, the experimentation of the method appeared useful to us. The urban sociological research used a sample selected from an area of Bucharest. For Electre we have considered three indicators related to: (1) dwelling; (2) the average time it takes one to get from home to his working place; (3) the average time it takes to supply oneself with food.

Indicators 2 and 3 have been expressed as time intervals, while indicator 1 has been defined by combining two indicators expressing density: m^2/person and rooms/person. In ordering the states, we have also resorted to a norm — according to some normative in force concerning the living space — which assigns a minimum surface sm^2 for each person.

Combining the three criteria, we have introduced a code with values from 1 to 9, which depend on the considered norm (sm^2).

The value of the importance coefficients of the criteria has been established with respect to the hierarchies suggested by the subjects in answer to the corresponding questions in the questionnaires.

In the case we deal with, the norm has been taken from an institutional act. However, this line of action is not compulsory. What is essential is the fact that the norm allows us to define more precisely the respective state and its relation to a value system. Obviously, as the norm changes in time and space, so do the code numbers of the individuals and the signification of the results.

The score interval, established by the Sturgess test, was about 3; a first variant of the results consisted in ten ordered groups; then, accepting the hypothesis of a normal distribution of data, the ten groups have been reorganized in five groups, the numbers of the groups being equivalent to the ranks in a hierarchy and the greatest number standing for the best situation. Our results have been synthesized in the following table, where we have replaced data by symbols.

The maximum code corresponds to the class (group) which has the best situation.

TABLE 3

Classes	Score interval		Frequency		Percentage	
1	$a - b_1$	5-10	A_1	46	B_1	19,08
2	$b_2 - c_1$	11-16	A_2	12	B_2	4,17
3	$c_2 - d_1$	17-22	A_3	51	B_3	21,12
4	$d_2 - e_1$	23-28	A_4	56	B_4	31,53
5	$e_2 - f_1$	29-35	A_5	76	B_5	31,53

Obviously, Electre may use many more data and — if a certain value system has been adopted — it may be used to organize the multi-dimensional values of some other social groups or populations, at a given moment, or the set of all states a certain population may achieve in time.

Electre is not the only method that may be used in the study of multi-dimensional concepts, the concept of quality of life included. Other methods may be used, even simultaneously, in order to compare the results.

A shortcoming of Electre — and of other methods as well — is its dependence on factors of the value systems. This happens when establishing the weighing coefficients of criteria or even when characterizing the states. As far as the latter aspect is concerned, we have suggested a device which allows a more accurate identification of the signification of the results. Certainly, both aspects may be re-examined. We may also wonder whether in dealing with certain problems we may avoid referring to weighting coefficients of the already-mentioned type.

The methods we mentioned have been examined from various points of view. Some are simpler and/or better known. Others are less known and/or more complex. Some of them — as those based on binary relations, pre-orders of k classes — have been considered inadequate for decision problems.

It is interesting to compare the results obtained by using more methods for studying the same problem. An example in this respect is given in (2); it refers to the ordering of seven projects concerning different possibilities of health assistance (the control of paludism or cholera, the training of cardiologists etc.). Three of the seven projects, numbered from one to seven, were to be chosen. The so-called weighting sum method established the order: 5, 1, 2, 4, 3, 6, 7, and Electre II 5 (1, 2), 4, 3, 6, 7. Thus the two methods led to the same choice: 5, 1, 2. If only one project has to be chosen, Electre I asks for a new investigation considering projects 5, 1, and 2, while the weighting sum method chooses project 5.

However, this does not answer the question whether more methods applied to the study of the same problems do or don't lead to identical results in other situations too. This problem, although important, is beyond our scope.

Electre — and not only it — should not be considered only from the point of view of its results. More definitions of the quality of life concept have been suggested. Most often their authors use results of certain observations and analyses, as well as elements suggested by desirable models of the quality of life. So the concepts seem rather bundles of dimensions and structures, while the relations between these dimensions are either omitted or vaguely perceived. In this context, Electre and other methods become clearly defined frames of reference, compelling the researcher to define more accurately his concepts and to become aware of the real possibilities of making them operational.

The case study presented above may be considered a methodological experiment, helping to reveal some important problems which may appear when applying the method to more numerous complex and representative data for the concept of quality of life.

If such a way of research is wanted, we cannot forget that sociological studies generally have a local character.

Electre should be considered a useful model of systematizing the analyses which have as objective the quality of life, alternative ways of life, alternative ways of development. The method serves the aims of system analysis.

Multidimensional States; Ways of Comparing Them

The possibility of comparing multidimensional states has been contested sometimes; however, they are still realities, noticed long ago, which have been the concern of many studies within the framework of various societies. In this respect I shall mention the results of certain investigations which make more precise some aspects of the problems we are dealing with.

I. Goldthorpe,[9] after pointing out that satisfaction with work could be successfully studied only in relation to a more profound question called orientation towards work, distinguishes among three such orientations: (1) instrumental; (2) bureaucratic, and (3) solidary.

Within the first orientation, work is mainly a means to one or more goals which are external to the work situations; the worker acts as a homo economicus, a major objective being to maximize income, to increase the purchasing power; work is rather "labour," "toil," it is not "part of the central life interests," the individual does not feel the "wish for a high level of satisfaction got from the common activity, in the relationships with his workmates," "the leaders' lack of orientation with respect to man" is not felt as distressing — yet "work and non-work are intimately linked together" and are part of a "whole way of life." For the instrumental type "his role in the family is prior to the production activity, to his role as a member of a club, a fellow." The instrumental type appears when the individual is selling his labour power in a typical market situation and the individuals "are compelled to a great degree to sell their labour in an instrumental way."

The individual's orientation to work, depending on social and cultural factors, on value systems, should be considered an essential mediator between "the features of the work situations, objectively considered," and the nature of the individuals' answers, "as a variable independent of the work situations."

The solidary-oriented human type is defined by different characteristics: he may sacrifice his economic goals when "the maximizing behaviour would alter the group norms and the group solidarity." Here we may speak about "an occupation/work community."

Trying to define by indicators the two types, we notice that one type gives "more" weight to some indicators and "less" to others, while the other type acts differently, sometimes in the opposite way.

According to their orientation, people will direct their attention to those "working places which offer the best opportunities for achieving the most valued recompenses." Obviously, the individuals compare and order multidimensional states and choose according to the hierarchies they establish. The measurements associated to these multidimensional states — considering one orientation or another — will express preferences for certain states.

In a more general frame, one way complexity makes itself evident in contemporary society is by confronting individuals with multidimensional situations more and more frequently. The individual has to compare, build hierarchies, and sometimes make a choice not between a good and a bad situation but between a bad and worse one. The situation of emigrant workers from the third world to industrially developed societies is only one of the well-known examples. The theory of the non-comparability of multidimensional states leaves aside the value systems of various individuals, social group, classes, the social theories; yet this operation itself depends on a system of values which are specific to the reductionist or econometric conceptions.

Needs, Satisfactions, Compensations

We are interested here in compensatory processes leading to compensatory models; they are expressed by a certain configuration of the levels of indicators and of coefficients of weighing indicators such that certain classes of multidimensional states seem to be preferable to others.

J. Galtung and M. Wemegah[7] identify a way of life which is dominant in high-income societies. This "dominant way" is associated to a certain compensatory model. The main elements of this model are: (a) the belief that the first aim is to consume and dispose of material objects, to have as much as possible; (b) the belief that the satisfaction — even over-satisfaction — of material needs may compensate or "pay" for the non-material needs, that is, for what we called the qualitative aspects of life. One could say that this is the internalized model of the economic exchanges with profit. "The dominant model" in high-income countries proposes such a compensatory model even to those that would prefer another one; the fact does not exclude, however, a great part of the population having no access to the conditions under which they "act" according to the respective compensatory model.

A compensatory model, once chosen, neither excludes nor automatically implies the existence of internal distortions among those who have adopted it. That part of the population that has "internalized" the model of the "dominant way" and has the necessary conditions to live according to it, may not feel any internal conflict. The individual who has adopted the "instrumental orientation" however, even accepting lack of satisfaction in his work, lives work as "toil."

The already mentioned paper of J. Galtung and M. Wemegah describes alternative ways of life opposed to the "dominant one" in high-income countries and points out that the ways of life should be in keeping with the convictions and value systems of those who have adopted them. The proposed model excludes inner conflicts within the individual.

Compensatory Models and Compensatory Methods

G. Păun[16] demonstrates that, under certain conditions, a mapping "g" aggregating more variable-indicators is compensatory in a certain sense. One has to show if (and which of) the already known methods of aggregating data are compensatory in that sense. One has to elucidate such problems as: (1) the way of defining g excludes dichotomic variables which may cause "catastrophic" variations, while Electre and other methods use dichotomic variables as well; (2) it is stipulated that the variables have values which may be codified by "little, significant, great, very great," and one has to establish the conditions under which the codification is to be used. For example, when the variable is the income, and there are two individuals having very different incomes, we have to refer to value systems, thus complicating the problem. Here however we shall disregard such observations and accept that Electre introduces some compensations — in the sense this concept has been defined in the previous section — or, to put it in other words, we consider the compensation of the low levels of certain indicators in the context of the high levels of other indicators and of certain weighting coefficients, such that a certain multidimensional state would seem preferable to another.

We are interested in a method having a compensatory character when asking if and/or under what conditions multidimensional social states may or may not be dealt with using compensatory methods.

Let us turn back in this context to the way of life which Galtung calls "dominating." A compensatory method may be applied from the point of view of this way of life. Let us accept that Electre is such a method; let us accept that we use indicators pointing to "qualitative" aspects as well. So, the coefficients of weighing the before-mentioned indicators would appear as insignificant when compared to those connected to material needs. By its compensatory character the method will give a distorted representation of the way various social groups are distributed, according to their level of the quality of life — distorted as compared to other views on the quality of life. Yet, we have to point out that the "orientations" — the value systems putting their stamp on the inputs of the method — will be the main source of distortions.

Another point of view may be formulated according to which weighting indicators is a matter of no importance since we have to admit that certain classes of indicators may be in compensatory relations with other classes. It is easy to give examples of social theories which distinguish groups and social classes in society, so that the attributes characteristic of one class — the disadvantaged one — measurable by indicators, may not be compensated in the context of the social structure by other attributes. We may say that such theories distinguish between groups, classes, or social facts which are in opposition.

In the light of such a theory, the shift from one group or class to another may not be codified by such categories as "little, significant, great, very great."

While at a very abstract level the dichotomic codes might be a kind of universal of social theories, of certain types of conceptions, their semantic — social, political, axiological — content is different from one theory to another. Moreover, some theories exclude the possibility of using methods which may compensate the dichotomic variable by other variables.

Simplifying, I shall admit that such opposition may be codified by dichotomic variables. The technical problem of codifying oppositions between social facts by dichotomic variables is a major one; however, it will not be dealt with here.

One of the facts which render the compensatory mathematical methods inapplicable to the study of multidimensional social states would be the dichotomic variables, more precisely a type of dichotomic variables which may codify certain fundamental oppositions between social facts. In their turn, the dichotomic variables express some models of dichotomic codification of social facts, certain dichotomic codes which in their abstract form are a common element, recurrent in the paradigms of various social, political, and religious theories.

So, what conclusions can we draw after identifying one of the major factors which make impossible the use of compensatory mathematical methods for problems such as those mentioned above?

First, new mathematical methods are needed, methods which should apply compensatory conditions to some classes of variables and non-compensatory to others, eventually "reflecting" the relations between the two. Speaking about measuring, comparing multidimensional entities, such as QOL, we should not only think of hierarchies, but also of some other models of "localizing" these entities in a multi-dimensional space. Such models should allow us to represent the opposition relations or relations of other types. Visualizing these results we could get representations similar to those obtained by using the methods of analysing "proximities," representations which may occur for example in R^2. The example is meant to give a more concrete support to the idea.

As for Electre, the above-mentioned dissociations were not necessary when applying it to the fields for which it has been conceived. The explanation is that such methods have been elaborated as instruments of the science of economic administration or of other particular problems and not within economy as part of the social sciences nor as instruments of social sciences.

However, by experimenting with the method we may get some information. At the applicative level, it may be used if a certain value system is adopted; when confronted with problems for whose solution it is not adequate, it proves to be stimulating for getting deeper into the structure of these problems.

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(Note: The operations of systematically collecting data and of processing them for the case study here presented were undertaken by the Laboratory of Sociological Studies and Researches, IPCT, Bucharest, 1977.)